STREAM GAGING

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BY

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"Much water goeth by the mill
That the miller knoweth not of."

—Epigrams and proverbs of
John Heywood, first printed in 1546.

PREFACE

The development of water-power, the building of impounding reservoirs, the irrigation of arid lands, and the carrying out of other hydraulic projects involving the use of streams, require the hydraulic engineer to have as a part of his professional equipment a ready and working knowledge of the general nature of stream-flow and the practical methods for determining the rate of discharge.

The purpose of this book is to present, briefly, such theories of stream-flow as bear on the subject of stream gaging, to consider practical means for applying these theories to the measurement of flow, to examine the characteristics of the various measuring devices, and to outline methods for the analysis of stream-flow data.

The development of stream gaging methods and the analysis of stream-flow data have been the special tasks of the engineers of the Water Resources Branch of the United States Geological Survey. From time to time, hydraulic engineers in the United States, as well as in other countries, have made individual investigations of problems related to the measurement and flow of streams. In the preparation of this book, there has been included much of the published information concerning the results of the investigations of these individual engineers and the work of the engineers of the Survey. It has been the intention of the author to make proper acknowledgement of the sources of information whenever he has drawn upon them.

The author wishes to make the following acknowledgements: to Mr. C. H. Pierce, formerly District Engineer, U. S. Geological Survey, Water Resources Branch, for reading portions of the manuscript; to Mr. H. B. Kinnison, District Engineer, U. S. Geological Survey, Water Resources Branch, Boston, Mass., for helpful suggestions and assistance in securing illustrations; to the U. S. Bureau of Standards, for the description of its rating station and equipment and for other data; to Mr. H. E. Hurst, Director General, Ministry of Public Works, Egypt, for supplying certain data; to Professor Dwight Porter, for per-

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W. A. L.

Cambridge, Massachusetts,
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STREAM GAGING

CHAPTER I

INTRODUCTORY

1. Nature of Stream Gaging.—Stream gaging is a subject which pertains, in general, to the measurement of the flow of water in open channels, and in particular, to the determination of the volume of water passing a given cross-section of a stream, at various gage heights, for the purpose of calibrating the stream.

Stream gaging operations consist in making such observations as are necessary to calculate the rate of flow. The method to be used and the data to be observed will be determined by such factors as the volume and velocity of flow, the type of channel, the facilities available, the degree of accuracy required, and the time involved.

2. Discharge.—The discharge is the volume of water passing a given section in a given time. It is equal to the product of an area, A, and a velocity, V, and is denoted by the letter Q. The area may be that of a single section of the channel or it may be the average of several sections, depending on the manner in which the velocity is determined. The velocity may be measured at a single section by some such instrument as a current meter, or it may be determined by the timing of floats over a course, or the measurement of the slope of the water surface.

If A is written in square feet and V in feet per second, then Q will be expressed in cubic feet per second, often shortened to "second-feet." The unit of discharge is the rate of discharge of water flowing in a channel of 1 sq. ft. cross-section and having a velocity of 1 ft. per second.

3. Stage.—The stage of a stream is the elevation of its water surface. In general terms, it may be referred to as high, medium, or low. Otherwise, it may be expressed in terms relative to some datum plane as indicated by a gage. The datum plane may be mean sea level, the deepest point in the channel, or any

other point to the elevation of which the elevation of the water surface is arbitrarily referred.

4. Stage-discharge Relation for Rivers.—The volume of water flowing in a stream will determine the height of the stage. At some points along the course of the stream, given volumes of flow will always cause the water surface to assume definite elevations. Elsewhere, for any given discharge, the elevation may vary from time to time and no indication of the rate of flow can be had from the gage reading. In the former case, the relation of the stage to the discharge is said to be constant and the gage reading may be relied on to indicate the corresponding rate of discharge from the stream. The constancy of the stage-discharge relation will be determined by various factors connected with the nature of the stream channel, a consideration of which is taken up in later articles.

5. Records Required for Stream-flow Studies.—In making studies for water-power developments, irrigation projects, municipal water-supply systems, and other undertakings involving the use of water from streams, it is desirable to have a record of the variations in flow for a period of years on which to base a prediction of the probable variation in flow in the future. This prediction is generally concerned only with the high, average, and low flow of the stream. For this purpose a continuous record of the daily average rate of flow of the stream is necessary. Such a record may also find a use in the operation of these hydraulic systems.

If the daily flow record were to be obtained by making discharge measurements each day, the task of such a procedure would be so great as to make the collection of the data extremely impracticable. If the discharge could be inferred from a gage reading, which would be a simple observation to make, a daily record of the discharge could be obtained without any difficulty. To infer a discharge from a gage height would require a constant relation between a given gage height and a given discharge. If such relations exist at all stages, the general relation of stage to discharge can be shown by means of a graph constructed from actual observations of discharge plotted against corresponding gage heights. With such a graph the discharge can be obtained for any given gage height.

6. Available Flow Records of Streams in the United States.—The usefulness of a daily record of discharge will depend on the

period of years over which it has been kept—the longer the period, the greater the usefulness. On some streams, especially the smaller ones, no records have been kept and, before making a study of such streams, it may be advisable to start such a record and keep it as long as possible before a final study of the stream is made.

On some streams, the records go back many years. At Lowell, Mass., records of the daily flow of the Merrimack River at that point go back to 1848. These records have been kept by the Proprietors of Locks and Canals on Merrimack River, a private company. On other streams, records have also been kept by water-power companies which furnish long-time flow data.

The bulk of the records has been obtained, however, by the U. S. Geological Survey which commenced its collection of stream-flow data in 1888. Its early records were obtained for the larger streams of the country. Gradually, new stations have been established so that, today, there are some 1600 stations located on the streams of the United States and its territories where daily records of flow at the several points are obtained.

7. Water Supply Papers.—The Survey publishes the records of the daily discharges obtained at its 1600 stations under the title of "Water Supply Papers." The data appearing in the Papers are assembled at Washington, D. C., from the records sent there by the various District Engineers located in different parts of the country.

These Papers take the form of annual reports and are divided into twelve parts, each part covering a major drainage area. The divisions are as follows:

I. North Atlantic.
II. South Atlantic and Eastern Gulf of Mexico.
III. Ohio River.
IV. St. Lawrence River.
V. Upper Mississippi River and Hudson Bay.
VI. Missouri River.
VII. Lower Mississippi River.
VIII. Western Gulf of Mexico.
IX. Colorado River.
X. Great Basin.
XI. California.
XII. North Pacific Coast.

MACHIAS RIVER BASIN

Machias River at Whitneyville, Maine

LOCATION.—At a wooden highway bridge in Whitneyville, Washington County, 200 feet below a storage dam and 4 miles above Machias.

DRAINAGE AREA.—465 square miles.

RECORDS AVAILABLE.—October 17, 1903, to September 30, 1918.

GAGE.—Chain installed on the wooden highway bridge October 10, 1911; prior to October 3, 1905, chain gage on the Washington County Railroad bridge, three-fourths of a mile downstream; October 3, 1905, to October 9, 1911, staff gage on highway bridge at datum of present chain gage. Gage read by I. S. Albee.

DISCHARGE MEASUREMENTS.—Made from railroad bridge or by wading.

CHANNEL AND CONTROL.—Practically permanent.

EXTREMES OF DISCHARGE.—Maximum stage recorded during year, 10.0 feet at 3 p. m. April 22 and 3.30 p. m. April 23 (discharge, 5900 second-feet); minimum stage recorded 3.25 feet on August 3, 4, 5, 6 and 7 (discharge, 160 second-feet).

ICE.—River usually remains open at the gage but ice farther downstream occasionally affects the stage-discharge relation.

REGULATION.—Opening and closing of gates in storage dam immediately above station each day during low stages of the river cause considerable fluctuation; some log driving every year and jams of short duration occasionally occur.

ACCURACY.—Stage-discharge relation practically permanent except when affected by ice. Rating curve well defined between 100 and 4000 second-feet. Gage read to tenths once daily, except from December 15 to March 30, when it was read three times a week. Daily discharge ascertained by applying mean daily gage height to rating table and making corrections for effect of ice during the winter. Records fair.

DISCHARGE MEASUREMENTS OF MACHIAS RIVER AT WHITNEYVILLE, MAINE, DURING THE YEAR ENDING SEPT. 30, 1918

Date	Made by	Gage height, feet	Discharge, sec.-ft.
Jan. 5	A. F. McAlary	[a]4.30	308
Feb. 16	A. F. McAlary	[a]5.1	538
Mar. 16	A. F. McAlary	[a]4.80	474
Aug. 11	H. A. Lancaster	4.23	640

[a] Stage-discharge relation affected by ice.

FIG. 1.—Illustrating the gaging station information which is contained in the water supply paper of the U. S. Geological Survey. (*From U. S. Water Supply Paper* 471.)

Daily Discharge, in Second-feet, of Machias River at Whitneyville, Maine, for the Year Ending Sept. 30, 1918

Day	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.
1	770	1380	598	360	270	860	920	4150	1380	490	244	178
2	980	1380	544	360	270	800	1250	4800	1380	387	200	221
3	860	1240	540	360	270	800	1860	3750	1380	387	160	221
4	711	1240	520	340	270	800	2200	3150	1240	387	160	314
5	654	1100	490	310	270	800	2500	2950	1240	387	160	362
6	711	1100	460	310	270	740	2660	2570	1100	438	160	412
7	860	1100	460	310	270	680	2750	2030	1100	860	160	412
8	860	980	440	310	270	660	2950	1860	1240	1700	200	412
9	980	980	440	340	270	640	2950	1540	1240	1540	200	412
10	980	980	410	360	270	580	3150	1540	1240	1380	490	412
11	1240	1240	410	360	270	520	2950	1540	1100	1380	654	412
12	1700	1100	410	360	270	490	2950	1860	1100	1100	682	412
13	2200	1100	410	360	270	490	2750	2030	1240	980	740	362
14	1860	980	410	360	270	460	2750	2390	1240	860	740	362
15	1540	860	410	360	390	460	2750	2570	1240	770	740	362
16	1540	770	410	360	540	470	2750	2950	1240	711	740	362
17	1540	711	410	360	520	490	2950	3150	1240	711	740	362
18	1540	711	410	360	490	520	2950	3350	1100	711	626	362
19	1700	711	390	360	490	520	2950	3350	1100	711	571	464
20	1860	711	369	360	490	540	2950	3150	1100	711	517	682
21	1860	711	360	360	490	580	2950	2750	1100	654	464	1380
22	1540	711	360	310	490	580	5900	2570	1100	598	412	1860
23	1540	860	360	290	490	580	5900	2390	1860	544	412	1940
24	1540	1100	360	270	520	600	5600	2210	1700	544	412	1940
25	2030	1100	360	270	580	640	5240	2030	1540	544	362	1460
26	1860	1240	360	270	640	640	4360	1860	1240	544	314	1240
27	1540	1380	360	270	740	660	3150	1700	1100	544	267	2120
28	1540	1380	360	270	860	680	3150	1700	860	490	221	3950
29	1540	1100	360	270	...	720	3550	1540	711	490	178	3150
30	1540	770	360	270	...	740	3750	1540	598	438	178	2750
31	1540	......	360	270	...	800	......	1380	......	338	178	

Note.—Stage-discharge relation affected by ice Dec. 3 to Apr. 5; discharge for this period computed from gage heights corrected for effect of ice by means of three discharge measurements, observer's notes, and weather records.

Monthly Discharge of Machias River at Whitneyville, Maine, for the Year Ending Sept. 30, 1918
(Drainage area, 465 square miles)

Month	Discharge in second-feet				Run-off (depth in inches on drainage area)
	Maximum	Minimum	Mean	Per square mile	
October	2210	654	1390	2.99	3.45
November	1380	711	1020	2.19	2.44
December	598	360	416	.895	1.03
January	360	270	325	.699	.81
February	860	270	411	.884	.92
March	860	460	630	1.35	1.56
April	5900	920	3180	6.84	7.63
May	4800	1380	2460	5.29	6.10
June	1860	598	1200	2.58	2.88
July	1700	338	720	1.55	1.79
August	740	160	396	.854	.98
September	3950	178	976	2.10	2.34
The year	5900	160	1090	2.34	31.93

Fig. 2.—Illustrating the gaging station records which are contained in the water supply papers of the U. S. Geological Survey. (*From U. S. Water Supply Paper* 471.)

At each gaging station, a record of gage heights, discharge measurements, and other supplementary information are obtained for the purpose of determining the daily flow. The discharge measurements are used in the preparation of tables giving the discharge at any stage. Daily, weekly, or monthly average discharges may be obtained by the application of gage heights to the table.

Generally these reports include a description of the station, a table showing the results of discharge measurements, a table showing the daily discharge of the stream, and a table of monthly and yearly discharge and runoff.

The description of the station includes such data as the location, equipment, nature of control, conditions affecting stage-discharge relation, and the accuracy of the records. Figures 1 and 2 are taken from U. S. *Water Supply Paper* 471, 1918, and illustrate the arrangement of the data as published by the Survey.

Water Supply papers are obtainable, gratis, from the Director of the Geological Survey, Washington, D. C., until the supply is exhausted, after which they may be obtained at a nominal charge from the Superintendent of Documents, Government Printing Office, Washington, D. C. A list of papers together with their prices may be obtained from that office.

8. Other Applications of Stream Gaging.—The methods developed for the gaging of streams are also applicable to the measurement of water for other than statistical purposes. For example, they have been used to measure flow in connection with the determination of friction factors for wood stave pipes, the effect of vegetation on the flow in canals, coefficients of roughness for various channel linings, sluice-gate coefficients, and other constants of a similar nature.

In the testing of hydraulic turbines and pumps, the rate of flow drawn by them has to be determined and some one of the several methods available for measurement can be used.

CHAPTER II

THE FLOW OF WATER IN RIVERS

9. General Nature of Flow.—It is essential that some recognition be given to the conditions under which water flows in open channels, particularly in rivers, and also that some knowledge be had of the laws governing the flow of streams before the methods to be used in the measurement of such flow are considered. The choice of the best sites for measuring and gaging stations, the proper type of gage installation, and the suitable measurement method to be used, together with the determination of the probable degree of accuracy obtainable, will depend upon an understanding of the various phenomena connected with the flow of the stream.

The occurrence of these phenomena will vary on different streams and, indeed, on any single stream it will vary from the source of the stream to its mouth. In the upper reaches of the stream, the amount of water to be measured will be small, gradually increasing as the mouth is reached. In a similar manner the slope of the water surface and the velocity of flow will change, the degree of ruggedness of the bed and banks will vary, the shape and area of the cross-section will differ, and in other ways will the nature of the stream be changed. Because of these progressive changes along the course of a stream, special attention has to be paid to the prevailing local conditions.

In the following articles of this chapter, the various phenomena of stream flow, together with their governing laws, are briefly considered in so far as they bear on the general subject of gaging. More extended considerations of the various phenomena may be found in texts on theoretical and applied hydraulics.

10. Early Studies of Stream Flow.—Galileo is the first name associated with the investigation of the flow of water in rivers. His studies were made during the years centering around 1610. Much earlier, a series of aqueducts, 250 miles in length, had been constructed for the purpose of bringing water to Rome. Such an undertaking is evidence of a possession of some knowledge of

hydraulic phenomena but very probably whatever knowledge the early Romans had, it was far from being scientific.

Galileo had little or no information concerning hydraulic laws on which to base his studies. In fact, Brunings quotes Galileo as saying that he had "found less difficulty in the discovery of the motions of the planets, in spite of their amazing distances, than in his investigations of the flow of water in rivers, which took place under his very eyes."

In 1628, Castelli, a student of Galileo, introduced for the first time the idea that velocity is an element in the movement of flowing water and concluded from his experiments that the velocity varied with the head. Another student of Galileo, Toricelli, in 1645, discovered and stated the fundamental theoretical hydraulic law that, "neglecting the resistances, the square of the velocity of water is proportional to the head of pressure, or in other words, flowing water follows the law of falling bodies."

With Toricelli's theorem as a basis, Guglielmini, at the close of the seventeenth century, developed the so-called parabolic theory of flow in rivers. According to this theory, the surface velocity would be a minimum and the bed velocity a maximum. As a matter of fact, the minimum velocity generally occurs at the bed and the maximum in the upper half of the depth.

In 1730, Pitot, after whom the Pitot tube is named, and in 1738, Bernoulli, who first stated the fundamental theorem of flowing water, also conducted theoretical investigations of the flow of water in rivers. Bernoulli, as a result of his studies, introduced gravity into the relation between the velocity and the head, stating that the velocity varied with the square root of the product of gravity and head. Brahms, about 1753, went a step further and stated that gravity was not the sole determinant of the velocity but, owing to frictional resistances, the ratio of the area of the cross-section to its wetted perimeter must also be taken into account. Later, Chezy expressed the theory of Brahms by the formula $V = C\sqrt{RS}$, which has served ever since as the basis of all flow formulas.

11. Beginning of Experimental Work.—It will be appropriate at this point to observe that, in the science of hydraulics, reasoning inductively from a theoretical basis alone is liable to result in a fallacy. Such reasoning must be consistent with the results of experience and experiments. The customary way to adapt theory to actual fact is to introduce into formulas certain coeffi-

cients or constants, determined from experimental studies, and, quite probably, no formula can be applied to any case concerned with the laws of natural phenomena without the use of such coefficients.

Until about 1764, abstract reasoning was the only basis of hydraulic formulas, but about that time Michelotti and Bossut expressed the conviction that such formulas must be obtained from the results of observations as well as abstract reasoning. Then follows the experimental work of Dubuat, Chezy, de Prony, Coulomb, Eytelwein, Brunings, Woltman, Funk, Darcy, Bazin, Ganguillet, Kutter, Cunningham, and others, all of which has contributed much to our present knowledge of the principles of the flow of water in open channels. For a detailed account of the results of their studies, reference may be made to standard textbooks of hydraulics.

12. Experimental Work Done in the United States.—The history of experimental work carried on in the United States, starts with that done by Captain A. A. Humphreys and Lieutenant L. H. Abbot in 1850 *et seq.*, on the Mississippi River, under the direction of the United States Government.[1] The immediate reason for this investigation was a certain distrust which the government's commission, organized for the purpose of studying the flow in the lower Mississippi River, had towards the formulas then in existence. The results of their studies, which were very thoroughly made, furnished important data dealing with the laws of flow in natural streams. It might be noted also that the results of their measurements were contrary to the theory of Guglielmini's, previously mentioned. The work of James B. Francis on the canals at Lowell, Mass., in 1855,[2] D. F. Henry, on the St. Clair River in 1869,[3] T. G. Ellis on the Connecticut River in 1870,[4] and G. A. Marr on the Mississippi River, at Burlington, Iowa, in 1879[5] added data of considerable value to stream gaging work and may, perhaps, be said to be the foundation upon which present day methods of stream gaging are laid.

[1] Humphreys and Abbot, "Report on the Mississippi River."

[2] Francis, "Lowell Hydraulic Experiments."

[3] *Journal*, Franklin Institute, vol. 62.

[4] *Report*, Chief Engineer, U. S. A., 1878, Appendix B.

[5] McKenzie, A., "Report on Current Meter Observations," Burlington, Iowa.

In 1888, the U. S. Geological Survey commenced the collection of stream flow data. This work involved studies of the various conditions of flow in rivers and much of the information available today, concerning the regimen of streams, is the contribution of these engineers.

In addition to the foregoing investigations, others have been made which deal with methods of measurement, behavior of instruments, comparisons of methods, and the accuracy of results, further mention of which will be made.

13. Complex Motion of Flowing Water.—By the early hydraulic experimenters, the motion of water in mass was assumed to be simple and analogous to the motion of a rigid body but later it was discovered that no indication as to the internal motion of the water was given by the motion of the mass of fluid as a whole. In fact, it has been proven conclusively by Osborne Reynolds that the motion in a mass of water may be of two kinds, one simple and the other complex. The first is known as steady motion, in which the motion of the particles at a fixed point is always constant. The second is known as unsteady or eddy motion, in which the motion at any point is always varying and in accordance with complex laws, being due to the formation of eddies or vortices in the liquid. It is this unsteady motion which is found in almost all cases of fluid motion coming up for practical solution. In the flow in open channels, this motion is extremely complex.

14. Causes of Complex Motion.—Where two adjacent streams of fluid are moving with different velocities, the common surface of separation between them is found to be in an unstable condition. This was demonstrated by Osborne Reynolds by using two liquids, carbon bisulphide and water, between which a horizontal surface of separation had been formed in a long horizontal tube. When the tube was tilted slightly so as to produce relative axial motion of the liquids, it was found that for extremely small values of the relative velocity, the motion was unstable. Reynolds determined from his studies of the motion of water that the conditions tending to instability and unsteadiness of motion are:

1. A decrease in viscosity.
2. Diverging solid boundaries.
3. Stream of fluid flowing through a fluid at rest.

4. Curvature, with the greatest velocity at the inside of the curve.

5. Greater density of fluid.

In streams, the water strikes obstructions along its course, such as stones, large rocks, sunken debris, depressions, etc., which interfere with the flow of water. This interference extends only to a relatively thin layer of water along the bed and the sides of the river. However, eddies are set up in the water due to the two adjacent layers of water, the bottom layer moving at a slow velocity and the upper moving at a faster velocity. Another source of eddies is the irregularity of the sides of the channel which cause the section of the stream to narrow and widen. At the point where the boundaries diverge, there is the condition of instability with resulting induced eddies. Again, the tendency toward instability occurs where there are two fluids of different density, as would be true in a degree near the surface of the water. The upper layer of water has a higher temperature than the lower layer with a correspondingly lower density at the top.

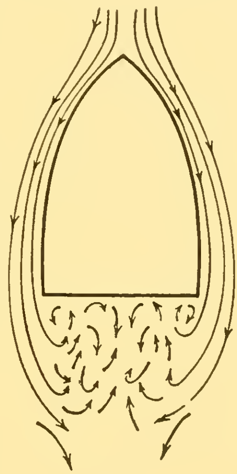


Fig. 3.

Eddies are also formed in the rear of immersed solids through the combined action of the water flowing around the edges of the solid and that drawn from its rear face. The formation of such eddies may be continuous or discontinuous. Figure 3 illustrates the action of the water in the rear of the object.

15. Translatory Motion of Eddies.—An eddy or a vortex is a mass of fluid which rotates about some axis in the fluid itself, forming a closed circuit. As formed in water, it will have a motion of translation through the surrounding water. This translatory motion of the vortex is not easily analyzed.[1] It is probably due to the difference in pressures at the inside and outside of the vortex. That at the outside, being the greater, causes a contraction of the vortex with a resultant change in conditions surrounding the vortex, both as to velocity and pressure. The net effect is to increase the pressure and decrease

[1] See a paper by W. M. Hicks in *Trans. Roy. Soc.*, 1884.

the velocity behind the vortex ring and to decrease the pressure and increase the velocity in front of the ring. The result is a translatory movement of the vortex through the water with an increasing velocity, limited only by the velocity within the ring itself. This movement of the vortex ring continues until its energy is spent.

16. Critical Velocities.—In the flow of water, there are two critical stages, so-called; a minimum stage, when the velocity is low enough not to develop any eddy action, and a maximum stage when the greatest retarding eddy action can be created. The point of maximum stage is determined by the roughness of the bed. Below the minimum stage and above the maximum, the tendency for the motion of the water is to be steady. For example, water flowing through a channel which is rough and has jagged surfaces, such as a cut through ledge rock, will flow, when the discharge is small and the slope of the bed flat, without any material disturbance of the water surface. As the discharge is increased, vortices will be formed, and boils and eddies will appear at the water surface. These eddies increase in turbulence until the maximum stage is reached. Above this point the boils and eddies at the surface gradually disappear. Eddying will still be observed, however, in the layers of water adjacent to the bed and sides of the channel. In the central portion of the stream, the stream lines will be practically steady. On the other hand, it has been observed that where the bed is sandy and wavy, the eddies occur at all times after the minimum stage has been passed.

17. Pulsation of Moving Water.—Another phenomenon which is found in the flow of water in both artificial and natural channels is the change in the velocity which is constantly taking place at any given point in the stream. This phenomenon is of special importance in its bearing on the method to be used in making a velocity determination and also the necessary time required for such a determination in order that the velocity may be accurately obtained.

Numerous observers have made investigations of this phenomenon on different streams and are in agreement as to the general occurrence of this pulsating effect in the water. These pulsations are probably due to the movement through the water of the eddies and vortices in such a way as to bring about a constant interchange of paths of currents moving at different speeds.

Much information concerning the period of duration and extent of the pulsations was obtained by L. C. Sabin on the St. Claire River in 1899. He used four meters placed 50 ft. apart and at the same depth. In one set of experiments, the line of meters was extended across the stream perpendicular to the axis of the stream, and in another set, the line was extended down stream parallel to the axis of the stream. Readings of the four meters were made simultaneously every 15 sec. for several periods of 10 min. each. When the observations were plotted with velocity against time, it was noted that there were two sets of waves, small ones of 15 to 60 sec. amplitude, and larger ones of 3 to 6 min. amplitude. In the latter case, a range of 35 per cent of the mean velocity was found in some cases.

There was a notable distinction between the curves drawn from the measurements made with the meters arranged in the transverse position and those drawn for the meters in the longitudinal position. In the former case, the curves would parallel each other at times and then intersect, which gave evidence that the pulsations did not extend across the stream. On the other hand, in the latter case, the curves ran along more nearly parallel, indicating that the pulsations can be traced for some distance in the direction of the thread of the current.

Among other instances where experiments have been made, the following may be cited: J. B. Francis[1] at Lowell, Mass., where the tube runs were made in a smooth wooden flume and the timing was done by means of an electic telegraph and chronograph, and the depth carefully measured by means of a hook gage. The variation in the velocity was between 8.57 per cent above the mean and 11.4 per cent below it. Professor Unwin[2] experimented with a current meter on the Thames River and found that the velocity as obtained from individual runs of 100 revolutions varied from the mean given by a continuous run of 1600 revolutions, by +8.3 to −6.0 per cent at 0.5 m. depth and +16.1 to −37.4 per cent at 6 m. depth. For a run of 100 revolutions, a difference may be obtained at 0.5 m. depth of 8 per cent, and at 6 m., 37 per cent. He also found, however, that when runs of 500 revolutions were considered there was a much smaller variation.

[1] *Trans. Am. Soc. Civ. Eng.*, vol. 7.

[2] *Proc. Inst. Civ. Eng.*, vol. 71.

D. F. Henry[1] found pulsations of different periods of amplitude occurring in both large and small streams, and also noted that the pulsations were smaller at the surface than at the bed of the stream. This difference between surface and bed fluctuations was also found by Harlacher,[2] varying in a few seconds from 20 per cent at the surface to 50 per cent near the bed. Experiments made by Marr on the Mississippi River at Burlington, Iowa,[3] with several meters read simultaneously showed a smaller range of fluctuation above and below the mean to occur at the surface than at the bed, though the changes in velocity occurred at about the same time.

From the foregoing it may be concluded that velocity observations taken during a short period of time will be of doubtful value. However, in practice, the length of time taken for a single measurement is from 35 to 70 sec. and to offset any inaccuracy, numerous measurements are made so that excessive velocities measured at some points will be compensated for by low velocities measured at other points.

18. Application of Filamental Theory of Flow.—The mathematical theory of flow assumes that the particles of water are flowing in parallel paths, and at steady velocities. From the foregoing articles, it is evident that such uniform filamental flow is not attained in rivers, although in artificial channels where the cross-sections are of constant shape and the lining reasonably smooth, uniform filamental flow may be closely approximated. Nevertheless, the filamental theory may be applied to the forward movement of the water and the effects of movements in other directions accounted for by coefficients for use in flow formulas.

The actual direction and velocity of a particle of a stream will be determined by such factors as gravity, the impact of other particles, its own energy, and any other force which can influence the motion of the particle. At any point, its instantaneous velocity may be resolved into three components one of which is vertical, and the other two transverse and parallel, respectively, to the course of the stream. As has been pointed out, the magnitude of these components will be constantly varying due to the phenomenon of pulsation but if we consider the flow of particles past

[1] *Journal*, Franklin Institute, vol. 62.

[2] *Trans. Am. Soc. Civ. Eng.*, vol. 12.

[3] McKenzie, A., *Report* on Current Meter Observations, Burlington, Iowa, 1884.

a given point in the stream over a period of time, the transverse components will balance, approximately, as will also the vertical components. The average of the longitudinal components will be the average forward velocity of the particle. These forward components may be considered as moving in straight lines and parallel with the paths of other particles and the filamental theory applied to the total forward movement of the water.

19. Laws Governing Flow.—From the foregoing it will be recognized that water flowing in a stream is subject to various laws, some of which are known but many of which are too intricate to permit of mathematical statement. It has been found, however, that the velocity of a stream is affected by (1) the frictional resistance offered by the bed and sides of the stream, together with other retarding forces, (2) the hydraulic radius, and (3) the slope of the water surface. The effect of the frictional resistance and other retarding forces, concerning whose laws little is known, is expressed in empirical formulas by means of a coefficient. The velocity has been found experimentally to vary very nearly with the one-half power of the hydraulic radius and the slope. Chezy, in 1775, proposed the equation $V = C\sqrt{RS}$, where C was a coefficient whose value depended on the effect of all factors other than R and S; R, the hydraulic radius; and S, the slope. The product of $A \times V$ is the discharge.

The hydraulic radius is the ratio of the area of the cross-section to the wetted perimeter. The length of the wetted perimeter will determine the amount of frictional resistance. Consequently, for a given slope of water surface and area of cross-section, the shape which gives the minimum wetted perimeter will give the maximum hydraulic radius and, in turn, the maximum velocity. For a river, the value of the hydraulic radius is continually changing due to the continual change in the area and shape of the channel sections, so that in the use of the Chezy formula for natural streams, an average value of R has to be used.

The surface slope is the primary cause of velocity. A level water surface would result in no flow but if the surface is higher at one point than at another, gravity will cause the water to flow from the higher to the lower level. The velocity is kept constant by reason of the friction of the bed and sides which just balance the accelerating force due to the slope of the water surface.

Chezy believed that the coefficient, C, varied with the degree of roughness of the channel lining only, and consequently, was

constant for all channels lined with the same class of material. Chezy did not give any values of C to be used in his formula, but various values have been proposed by later experimenters. Some of them are the following: Young, 84.3 for large streams; Eytelwein, 93.4; D'Aubisson, 95.6 for velocities over 2 ft. per second; Leslie, 68 for small streams and 100 for large ones. Later experimenters have concluded that C is also a function of R (and in some cases, R and S), as well as a coefficient of roughness. Formulas have been derived to show the relation between C and these several factors and are given in a later chapter on slope measurements.

The Chezy formula has not been the only formula proposed for determining the velocity of flow. Others of more complicated form have been derived but have never been generally used. They all have included the factors R and S, and have allowed for the frictional effect of the lining of the channel.

In recent years, a formula of the form, $V = KR^xS^y$, has been investigated. The exponents x and y of R and S, respectively, are not $\frac{1}{2}$, as is true in the Chezy formula, but have such values as will make the coefficient K constant for any channel lining.

20. Transporting Power of Flowing Water.—The importance of a knowledge of the transporting power of flowing water is not as great in connection with the gaging of streams as with their regulation, but where streams flow through soft alluvial lands, and are subject to considerable shifting of their beds, such knowledge will aid in choosing a suitable site for a gaging station.

The material which is transported by the stream is moved either in suspension or by rolling along the bed of the stream. A solid when immersed in still water will settle to the bottom under the action of gravity. When immersed in flowing water it will tend to settle gradually, but due to the velocity of the water, will be carried down stream. The downward movement of the solid particle will be opposed by upward components of the complex currents originating along the bed of the stream. The effect of the upward components of these currents is felt in the upper half, but to a less extent than in the lower half, so that only the finer particles are held in suspension in the upper layer. The heavier particles will ultimately reach the bottom and then be rolled along the bed of the stream by the force of the longitudinal velocities.

The transporting power of the water will increase with the velocity. For the finer particles, such as clay, the suspension may be continuous at all velocities. For the coarser particles the suspension may be intermittent, depending upon the velocity and whether they are suspended or rolling. For alluvial streams, the proportion carried in suspension is probably greater than that which is moved by rolling. When a solid is deposited on the bed it will require a greater velocity to erode it than to carry it in suspension.

The amount of suspended material, per unit volume, will be greater at the bottom than at the surface because of the stronger upward currents at the bottom than at the surface.

21. Silting and Scouring.—Silting and scouring are dependent on the velocity and the depth. Kennedy[1] has stated that the velocity at which neither silting nor scouring will occur is given by the equation.

$$V_c = cd^{0.64}$$

where V_c is the "critical velocity" or the velocity at which silting is just prevented, and d is the depth.

The coefficient, c, will have different values depending on the nature of the material carried by the stream. Values suggested by him are the following:

$c = 0.82$ for fine, light, sandy silt.
$c = 0.90$ for coarser light, sandy silt.
$c = 0.99$ for sandy loam.
$c = 1.07$ for coarse silt.

The capacity to transport materials varies approximately as the five-halves power of the velocity[1] and the weight of solids moved varies as the sixth power of the velocity. Consequently, small changes in the velocity will cause relatively large changes in the carrying capacity of the stream. Therefore, silting may be expected at any point where the velocity is reduced, as for example, the point where a stream widens or where there is an obstruction which slows up the current. Such points should be avoided in selecting the gaging site.

The velocity at which scouring will occur depends upon many factors such as size, shape, and uniformity of materials, volume of discharge, and width and depth of the channel. Values of scouring velocities for different materials can at best give but an

[1] *Proc. Inst. Civ. Eng.*, vol. 119.

indication as to the safe velocities to be used. Ganguillet and Kutter suggested the following values.

Material	Safe Bottom Velocity Feet/Second
Soft earth	0.25
Soft loam	0.50
Sand	1.00
Gravel	2.00
Pebbles	3.00
Conglomerate	5.00
Stratified rock	6.00
Hard rock	10.00

22. Effect of Bends.—It has been observed that where the alignment of the stream is straight, the flow may be considered as stream line and uniform. When a bend is encountered the regularity of the flow is disturbed. Due to the centrifugal effect of the water flowing around the bend, the elevation of the water surface will be raised at the outer bank and lowered correspondingly at the inner bank. The surface will then have a transverse slope as well as a longitudinal slope.

Because of this difference in level at the outer and inner banks, and the frictional effect of the bed on the outward bed velocity, causing it to be less than the outward velocity at the surface, a reversal in the transverse flow at the bed takes place. This transverse inward flow at the bed and the motion of translation downstream combine to give a helicoidal motion to the flow.

This helicoidal motion was demonstrated by Prof. James Thompson before the Institution of Mechanical Engineers of Glasgow in 1879. Leliavski made some observations at bends in the Dnieper River which were described in a paper presented to the Sixth International Navigation Congress held at the Hague in 1894. Leliavski concluded from his experiments that in a bend the surface currents move convergingly toward the outer bank, then to the bottom and, in diverging paths, along the bed to the inner bank and then rise slowly to the surface. Figure 4 illustrates the paths of the several velocities. The dotted lines represent the paths of the converging velocities along the surface and the full lines the paths of the velocities along the bed. The section through *MN* shows the direction of velocities in the cross-section.

The effect of this action of the water is to produce eddyings in the water and to cause a change in the cross-section due to the scouring action of the velocity at the outer bank and the silting action at the inner bank.

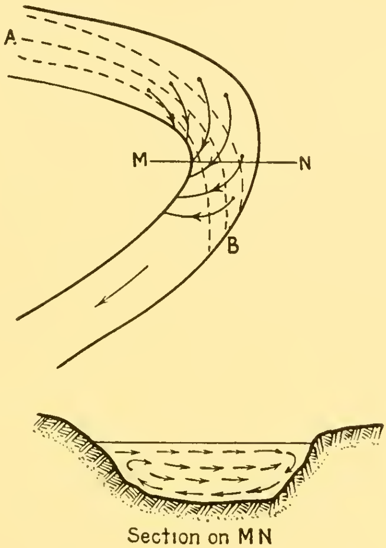


Fig. 4.

Moreover, the effect of the bend is not merely local but is felt in the straight sections of the stream for a considerable distance above and below the bend. The effect above the bend is the production of back water due to the necessarily gradual change in the transverse slope of the water surface. After the helicoidal motion is developed, some distance is required to allow this motion to be transformed into its steady filamental flow.

CHAPTER III

DISTRIBUTION OF VELOCITIES IN OPEN CHANNELS

23. General Distribution of Velocities in Cross-section.—Although the flow in open channels is complex and subject to continuous pulsations, certain general characteristics of the distribution of the velocities in a cross-section may be observed.

The highest velocities will be found, in general, in the portion of the section containing the deepest part of the channel. Due to the retarding effect of the sides, the velocities will gradually decrease towards the sides. This variation from a maximum

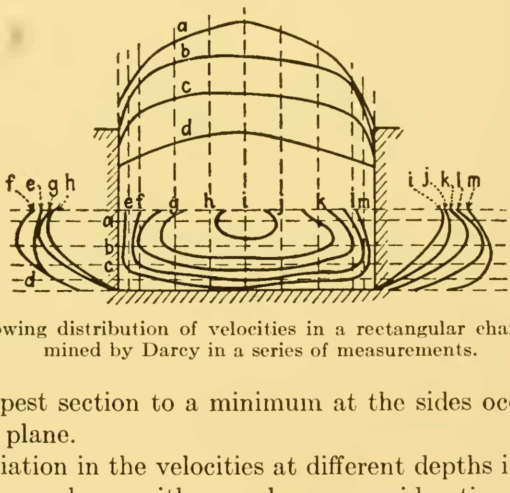


Fig. 5.—Showing distribution of velocities in a rectangular channel as determined by Darcy in a series of measurements.

at the deepest section to a minimum at the sides occurs in any horizontal plane.

The variation in the velocities at different depths in a vertical will be in accordance with some law, a consideration of which is given later. More definite knowledge may be had of the vertical variation than of the horizontal variation.

Points of equal velocity generally occur in such a way that curves of equal velocity may be drawn through them. Such curves would correspond to the contour lines of a topographical map.

Figure 5 shows the results of some gagings made by Darcy in a rectangular channel 0.25 m. deep and 0.8 m. wide. It illustrates the general distribution of velocities in a rectangular chan-

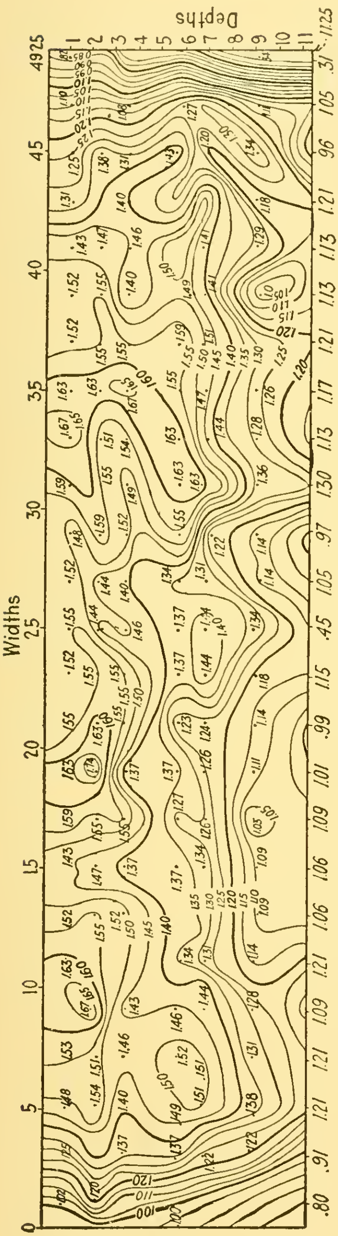


Fig. 6.—Distribution of velocities in measuring flume, Merrimack Canal, Lowell, Mass.

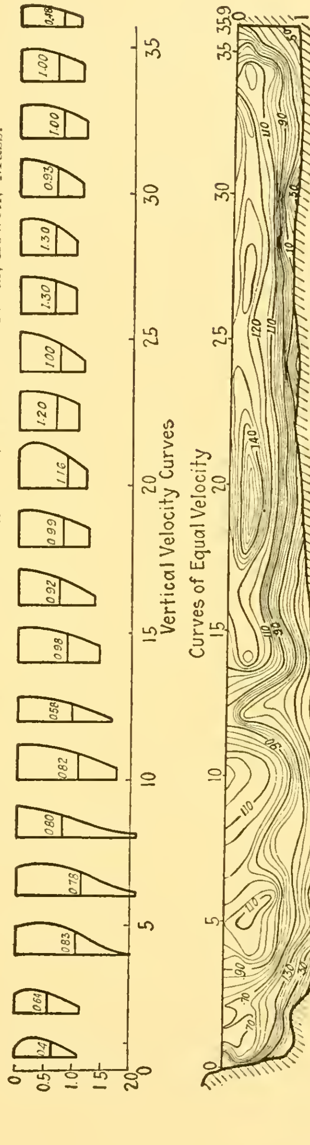


Fig. 7.—Distribution of velocities at M. I. T. gaging station, Chase Mill Stream, E. Machias, Me.

nel showing curves of (1) equal velocities, (2) variation of velocity in several verticals, and (3) variation in velocity in horizontal planes across the channel.

Figure 6 shows curves of equal velocity for a rectangular channel 49.25 ft. wide and 11.25 ft. deep. The irregularity of these curves illustrates the complex distribution of velocities in an open channel. Figure 7 shows curves of equal velocity and variation in velocities in verticals taken every 2 ft. across the stream. This stream was about 36 ft. wide and varied from a depth of 1 ft. to 2.2 ft.

24. Position in Vertical of Filament of Maximum Velocity.—In itself, the position of the thread of maximum velocity is not of great importance. However, the position of the thread of mean velocity is governed largely by the position of the thread of maximum velocity, therefore, it will be useful to have some idea as to the range of depth in which the maximum velocity occurs, and also what factors influence its position.

The particles of water flowing past a vertical would have equal velocities throughout its entire depth if it were not for the fact that the particles are retarded by certain frictional forces which vary in magnitude from the surface to the bed. These forces are greatest near the bed, their effect gradually diminishing as the surface is reached. At the surface the effect of these retarding forces should be a minimum and the maximum velocity might be expected to occur there.

Early measurements of the maximum velocities were obtained by means of surface floats, the velocity of the float being considered as the maximum velocity in the vertical. Later experiments showed that the occurrence of the maximum velocity at the surface was only true where the stream was broad and shallow and also had a high velocity. In other cases, the maximum velocity was found to occur generally below the surface at points varying from just below the surface to mid-depth.

25. Effect of Air Resistance on Depth of Maximum Velocity.—This retarding of velocities at the surface may be due in part to the resistance of the air. Major Allan Cunningham who carried on extensive experiments at Roorkee on the Ganges Canal[1] was a notable proponent of this idea. He considered the air as an efficient resisting factor and, as such, would cause the filament of flow to be everywhere depressed. But, allowing for the fric-

[1] *Proc. Inst. Civ. Eng.*, vol. 71.

tional effect of the air (the unit resistance is, roughly, one-tenth the resistance between the water and the channel lining), it seems probable that the resistance of the air is not the principal cause of the depression of the filament of maximum velocity.

26. Effect of Channel Friction on Depth of Maximum Velocity. In addition to the forward velocity of the stream there is a well-defined upward current along and close to the sides. This current is naturally slow because of the resistance offered by the channel lining. On reaching the surface, these upward currents are transformed into surface currents and move toward the center combining with other surface currents. The net result of this is to retard the surface velocity and cause the maximum velocity to occur below the surface.

F. P. Stearns[1] demonstrated this upward movement of water along the sides by placing a 6-in. board vertically in the current at the side of a rectangular channel 19 ft. wide. This obstruction made the upward movement more apparent. Sawdust was mixed with the water to make the visibility of the flow of the water better and it was observed that the water flowed from below, upward along the up-stream face of the board to the surface, and then moved obliquely toward the centre of the stream. Mr. Stearns suggested as a reason for the upward movement of the water that the obstructions along the sides retarded the velocity of the water striking them, thereby setting up an excess of head in a small pyramid of water just above the obstacle which, in turn, set up transverse velocities in all directions, the greatest being in the direction of least resistance, which would be upward.

Another explanation of this upward movement of the water along the sides of the channel is that of Gibson.[2] The lowering of the velocity along the sides reduces the energy, and consequently the head, of water adjacent to the sides. This has the effect of making the cross-sectional profile of the water surface concave toward the bed, with the central portion higher than that at the sides.

Due to this state of unstable equilibrium, the water at the centre tends to find its own level with the result that transverse currents are set up, which pass downward at the centre, outward along the bed, upward at the sides, and thence inward along and near the surface. These inward currents at the surface, having

[1] *Trans. Am. Soc. Civ. Eng.*, vol. 12.

[2] Gibson, "Hydraulics and Its Applications," D. Van Nostrand Company

come from the sides, are slow and, consequently, will have the effect of retarding the surface velocity and depressing the filament of maximum velocity.

This action of induced currents, so to speak, is shown by the diagrams *A* and *B* in Fig. 8, where *s* and *b* show, respectively, the directions of the transverse currents in the surface and bed of the stream.

These two theories differ simply in the explanation as to the direct cause of the upward currents, being in agreement as to the cause of the depression of the maximum velocity, namely, roughness of the sides.

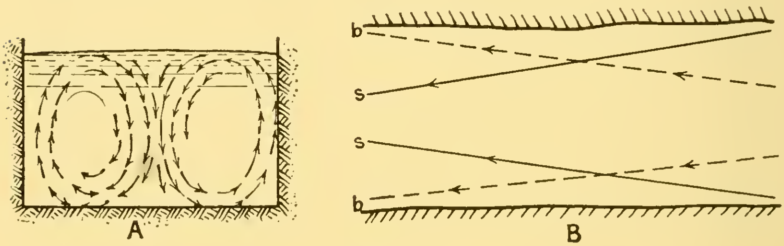


Fig. 8.

From both theory and practice it is found that the depth of the filament of maximum velocity in any vertical is:

1. Lower, the rougher the channel sides.

2. Lower as the influence of the sides increases, or where the ratio of width to depth decreases.

3. Lower as the roughness of the bed decreases because the retarding effect of the bed on the transverse current is not compensated in any way.

4. Lower near the sides of a rectangular channel.

27. Mississippi River Investigations.—In the work of Humphreys and Abbot on the Mississippi River,[1] observations were made of the position of the thread of maximum velocity in 69 verticals as obtained by 297 runs. The results are shown in the following table:

[1] Humphreys and Abbot, *Report* on the Mississippi River.

TABLE I.—TABLE SHOWING POSITION OF THREAD OF MAXIMUM VELOCITY OBTAINED BY HUMPHREYS AND ABBOT ON THE MISSISSIPPI RIVER

Depth	Number of verticals	Number of floats
Surface	20	80
1/10	12	48
2/10	5	33
3/10	4	23
4/10	4	9
5/10	3	25
6/10	6	33
7/10	6	18
8/10	6	15
9/10	2	5
Bottom	1	8
Summary	69	297

From this table it would appear that there is considerable variation in the position of maximum velocity in a vertical, and that measurements made at the same depth in a series of verticals cannot be indiscriminately grouped together for the purpose of obtaining the average velocity at that depth.

28. Cornell University Investigations.—Investigations made at Cornell University[1] furnish additional information with regard to the occurrence of maximum velocity in the vertical. The canal was concrete lined and 16 ft. wide, the depth varying for different experiments. The series of experiments can be divided into five groups, each of which had certain characteristics as follows:

Group A.—Comparatively deep, small range of depths, small range of velocities, smooth water surface, no eddies, sensibly stream-line flow.

Group B.—Shallow depths, high velocities, surface quite rough, being a succession of waves but without any eddyings.

Group C.—Depths ranged from 6 to 9.5 ft., velocities from 0.23 ft. per second to 2 ft. per second, smooth water surface, again a succession of waves without boiling or eddying.

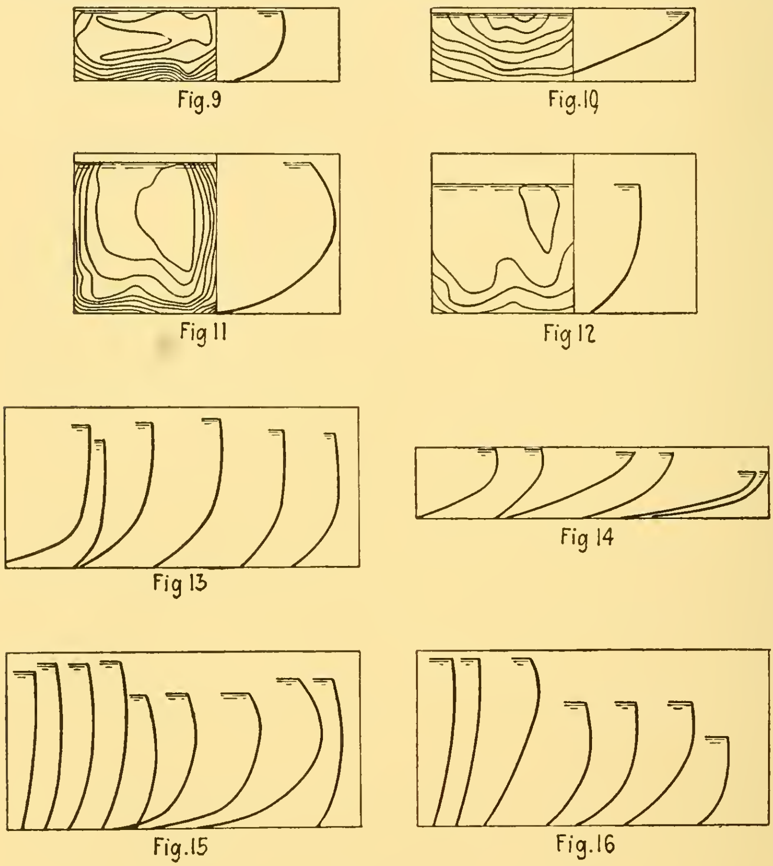
Groups D and *E* were similar to *C* and will not be further described.

Figures 9, 10, 11, and 12 show the curves of equal velocity and a typical vertical velocity curve for groups *A*, *B*, *C*, and *D* respectively.

[1] *U. S. Water Supply Paper* 95.

In Figs. 9, 11, and 12 the maximum velocity is below the surface and is not midway between the banks, being less at the center than at 3 or 4 ft. from the sides. In Fig. 10 the maximum velocity is in the surface and midway between the banks.

Figures 13, 14, 15, and 16 show typical vertical velocity curves obtained in groups *A* to *D*, respectively. The shape is influenced



Figs. 9–16.

evidently by the depth and the velocity. For low depths, say up to 1 ft., the curve is nearly a straight line with the maximum velocity occurring in the water surface and the mean velocity at middepth. For depths of 8 to 9 ft. and with small velocities, the curve is flat, and the maximum velocity occurs at two-tenths

to three-tenths the depth and the mean velocity at 0.65 to 0.75 depth below the surface. When the velocity is increased, the radius of curvature is less and the maximum velocity occurs at a lower depth. In all cases, the minimum velocity occurs at the bottom.

The results, as found from the series, may be tabulated as follows:

TABLE II.—SUMMARY OF RESULTS OF CORNELL UNIVERSITY INVESTIGATIONS OF VELOCITY DISTRIBUTION

Group	Position of maximum velocity (from surface)	Position of mean velocity (from surface)	Position of minimum velocity
A	0.18*D*	0.66*D*	Bottom
B	0.01*D*	0.54*D*	Bottom
C	0.31*D*	0.66*D*	Bottom
D	0.24*D*	0.63*D*	Bottom

It may be concluded from these experiments that, in general, where the depth is less than 2 ft., the thread of maximum velocity will be found in the surface of the stream provided there is no obstruction to the flow below the section. For depths of 5 ft. and more, the position of the thread of maximum velocity is lowered and may occur in the range of from zero to four-tenths the depth depending on the nature of the river channel below the section. Any obstruction will tend to lower the position.

29. Vertical Velocity Curve.—It has been observed in the foregoing that, in general, the maximum velocity will occur at or near the surface, say, the upper one-third of the depth, and the minimum at the bed of the stream. The velocities in between these extremes will be considered now, and, to simplify matters, it will be supposed that the velocities are obtained in a vertical at every one-tenth of the depth from the surface to the bed and that these velocities are obtained simultaneously and the time taken to obtain them sufficiently long to average the pulsations. If the velocities thus obtained are laid off to scale from a vertical line representing the vertical in which the velocities were measured, they will plot so that a smooth curve can be drawn through them. This curve is generally styled the vertical velocity curve and shows the manner in which the velocity varies from the surface to the bed of the stream.

It was stated that these measurements were made simultaneously. This is the ideal method, for the points on the curve are then obtained at one observation and the time required for a vertical is relatively short. Of course, such a method will require the use of a considerable number of meters, which will make the measuring outfit rather expensive. The more common method, perhaps, is to use a single meter and hold it at the different depths. It would take, roughly speaking, ten times as long as the multiple meter method and would have the disadvantage of requiring so much time that an important change in the mean velocity itself may occur. However the resulting curve will generally be good enough for practical purposes.

30. Shape of the Vertical Velocity Curve.—The geometrical shape of the vertical velocity curve is of some importance for a knowledge of its shape may lead to the discovery of important rules and limitations to be considered in the selection of points for velocity measurements.

The investigations as to the shape of this curve date back to 1791 when Woltman, after making some experiments on the Rhine, concluded that the vertical velocity curve was a reversed parabola with its vertex below the river bed. In 1820, De Fontaine, from his experiments, concluded that the curve consisted of two inclined right lines, intersecting at mid-depth. Also in 1820, Funk used a logarithm table in working up his observations on the Weser River. In 1824–1826, Racourt stated that the curve was an ellipse whose minor axis is a little below the surface, basing his statement on his observations on the Neva River; in 1844, Boileau determined from his work on the flow through a small canal that the curve was a parabola with its axis near the surface: Darcy and Bazin found it to be a reversed parabola, the parameter changing with the character of the bed; and Humphreys and Abbot, on the Mississippi, found it to be a parabola whose axis was horizontal and three-tenths the depth below the surface. This parabola with the horizontal axis was also observed by Cunningham and Ellis in their experiments.

In view of the foregoing, the shape of the curve may be said to depend largely on the local conditions, such as the roughness of the channel, the velocity of the stream, the width, depth, etc. When the velocities are plotted, the chances are that the points will not fall on a smooth regular line and it will be possible to

pass several curves through these points, any one of which will satisfy the observations as well as another. However, it is now generally accepted that the curve found by Humphreys and Abbot, namely, a parabola with a horizontal axis, most nearly expresses the relation between the velocity and the depth.

31. Mathematical Demonstration of the Shape of the Curve.—A theoretical analysis of the shape of the vertical velocity curve may be made by using Bovey's demonstration[1] as a basis. The equation derived will be based on certain assumptions, and of course, such an equation will apply only when the assumptions are justified. In other cases, the equation will serve as a basis for a more exact formula. The following assumptions are made:

1. The ratio of width to depth is large.
2. The depth is sensibly uniform.
3. The flow is sensibly steady.
4. The flow is sensibly stream line.
5. The resistance to flow is entirely due to viscous shear. (This assumption is perhaps the least tenable as it is never exactly realized.)

In Fig. 17 let

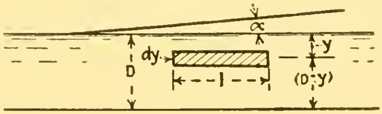


Fig. 17.

y, be the depth of a prism of water, below the water surface,
dy, the thickness of the prism,
l, the length of the prism,
b, the width of the prism,
w, unit weight of water,
v, velocity of water at depth y.

The forces acting on this prism in the direction of motion are:

1. End pressures.
2. Component of weight.
3. Viscous shear on two lateral faces.
4. Viscous shear on upper and lower faces.

End Pressures.—Since the end areas are equal and at the same depth below the water surface, the pressures are equal and neutralize each other.

Component of Weight.—Weight, $W = wbl \cdot dy$. (1)

[1] See "A Treatise on Hydraulics" by Henry T. Bovey, John Wiley & Sons.

Component in the direction of flow $= wbl \cdot dy \cdot \sin \alpha$ where α is the angle the surface makes with the horizontal.

Viscous Shear on Two Lateral Faces.—This is negligible in a stream of indefinite width, since there will be no relative sliding.

Viscous Shear on Upper and Lower Faces.—This will be proportional to the shear per unit of time and is measured by shear per unit of area at the actual rate of shearing.

The coefficient of viscosity, m, or simply the viscosity, is the ratio of the shear per unit of area to the shear per unit of time and defines that quality of the fluid by virtue of which it resists a change of shape.

By Navier's hypothesis, the viscous shear along the top surface is:

$$-mbl\frac{dv}{dy}. \tag{2}$$

Along the bottom face it is:

$$mbl\frac{dv}{dy} + mb \cdot ld\left(\frac{dv}{dy}\right) \tag{3}$$

$$= mbl\frac{dv}{dy} + mbl\frac{d^2v}{dy^2}dy. \tag{4}$$

The algebraic sum of the several forces being zero, since the motion is uniform, we may write,

$$wbl \cdot dy \cdot \sin \alpha - mbl\frac{dv}{dy} + mbl\frac{dv}{dy} - mbl\frac{d^2v}{dy^2}dy = 0,$$

which reduces to

$$\frac{w \sin \alpha}{m} + \frac{d^2v}{dy^2} = 0, \tag{5}$$

or

$$\frac{d^2v}{dy^2} = -\frac{w \sin \alpha}{m}.$$

Integrating twice,

$$v = -\frac{w \sin \alpha}{2m}y^2 + By + C. \tag{6}$$

When $y = 0$, $v = v_s$ (surface velocity) and $C = v_s$

therefore

$$v = -\frac{w \sin \alpha y^2}{2m} + By + v_s, \tag{7}$$

$$= v_s - \frac{w \sin \alpha}{2m}\left[y - \frac{Bm}{w \sin \alpha}\right]^2 + \frac{B^2m}{2w \sin \alpha}. \tag{8}$$

This equation is that of a parabola having its horizontal axis at a depth,

$$\frac{Bm}{w \sin \alpha} = y_1 \tag{9}$$

below the water surface.

Since the maximum velocity will occur at y_1,

substituting $\frac{Bm}{w \sin \alpha}$ for y in Eq. (8),

$$v_{(\max)} = v_s + \frac{B^2 m}{2w \sin \alpha}. \tag{10}$$

From Eq. (9)

$$B = \frac{y_1 w \sin \alpha}{m}.$$

Substituting this value of B_2 in Eq. (10),

$$v_{\max} = v_s + \frac{y_1^2 w^2 \sin^2 \alpha m}{2m^2 w \sin \alpha}$$

$$= v_s + \frac{y_1^2 w \sin \alpha}{2m}. \tag{11}$$

Adding Eqs. (7) and (11) and eliminating B,

$$v + \frac{w \sin \alpha y^2}{2m} - \frac{y y_1 w \sin \alpha}{m} = v_{\max} - \frac{y_1^2 w \sin \alpha}{2m}.$$

$$v_{\max} - v = \frac{w \sin \alpha}{2m}[y^2 - 2yy_1 + y_1^2]$$

$$= \frac{w \sin \alpha}{2m}[y - y_1]^2. \tag{12}$$

In a similar manner it may be shown that

$$v_{\max} - v_b = \frac{w \sin \alpha}{2m}[D - y_1]^2.$$

Mean Velocity.—From Eq. (12) the mean velocity may be obtained by integrating the sum of such terms as vdy over the vertical and dividing the result by the depth, d, of the vertical, thus:

$$v_{\text{mean}} = \frac{\int_o^D \left[v_{\max} - \frac{w \sin \alpha}{2m}(y - y_1)^2 \right] dy}{D}$$

$$= v_{\max} - \frac{w \sin \alpha}{2m}\left[\frac{D^2}{3} - Dy_1 + y_1^2\right]. \tag{13}$$

To find the depth of the mean velocity add Eqs. (12) and (13), thus

$$\frac{w \sin \alpha}{2m}\left[\frac{D^2}{3} - Dy_1 + y_1^2\right] = [y_1 - y_m]^2.$$

$$\therefore y_m = y_1 \pm \sqrt{\frac{D^2}{3} - Dy_1 + y_1^2}. \tag{14}$$

From this consideration, the position of the thread of mean velocity is shown to be a function of the depth of the thread of

maximum velocity, and consequently due attention must be given to the conditions which affect the position of the thread of maximum velocity before deciding upon the point at which the mean velocity can be measured directly. The conditions and their effect upon the position of maximum velocity have been discussed in the foregoing pages.

32. Experimental Determination of Position of Mean Velocity.—As a part of the experiments conducted at the Cornell Laboratory, previously mentioned, the position of the thread of mean velocity was observed. The results of the four groups of measurements are shown in Table II. This table shows that the mean velocity was found to occur in the vicinity of six-tenths the depth, being highest when the maximum velocity was 0.01 the depth and lowest for the deepest position of the maximum velocity.

From observations[1] made by the U. S. Geological Survey on many rivers in the United States the ratio of the mean velocity to the velocity at six-tenths the depth ranged from 0.94 to 1.04, with a mean of practically 1.00.

In 1894, a series of experiments were made on the Merrimack Canal at Lowell, Mass., by Wheeler and Lynch[2] for the purpose of investigating the character of velocity curves in canals. The canal used by them was about 48 ft. wide and 10 ft. deep with velocities of between 3 and 4 ft. per second.

As a result of some eighty odd individual sets of measurements, the average relative depth of the thread of mean velocity was found to be 0.67.

The following table shows a portion of the results obtained by the U. S. Geological Survey from measurements made on certain rivers in New York, Virginia, Tennessee, North Carolina, and South Carolina. The purpose of the measurements was to obtain data bearing on the mean velocity in a vertical.

[1] Hoyt and Grover, "River Discharge," John Wiley and Sons.

[2] See *Thesis* of Wheeler and Lynch, C. E. Dept. M. I. T., 1894.

TABLE III.—SHOWING RESULTS OF VERTICAL VELOCITY CURVE MEASUREMENTS MADE BY U. S. GEOLOGICAL SURVEY

River	Approximate width in feet	Range of depth in feet	Coefficient for reducing to mean velocity in any vertical the velocity observed at the following points			Per cent of depth at which thread of mean velocity is found
			$\frac{6}{10}D$	Mean top and bottom	Top	
James, Va.	850	3.0 – 4.4	1.01	1.08	0.88	59.6
Saluda, S. C.	800	3.0 – 8.5	0.97	1.03	0.82	62.2
Little Tennessee, N. C.	660	3.6 – 6.0	1.03	1.06	0.83	58.7
Dan, N. C.	600	1.4 – 3.7	0.99	1.01	0.86	61.6
Broad, S. C.	500	5.0 – 8.9	0.96	1.22	0.89	65.4
Yadkin, S. C.	450	4.0 –11.3	1.02	1.06	0.81	59.2
Staunton, N. C.	400	2.0 – 6.0	0.96	1.12	0.95	65.7
Wateree, S. C.	300	12.3 –17.7	1.01	1.13	0.90	58.4
Nolichucky, Tenn.	300	1.6 – 5.1	1.02	1.02	0.80	59.1
Catawba, N. C.	200	1.8 – 5.0	1.00	1.04	0.82	60.6
Catawba, N. C.	200	6.3 – 7.0	1.00	1.08	0.88	58.5
Appomatox, Va.	150	2.5 – 4.9	1.00	1.11	0.81	61.5
Yadkin, N. C.	160	2.7 – 6.3	0.96	1.12	0.91	66.5
Esopus Creek, N. Y.	100	2.0 –14.0	1.00	....	0.88	61.7
Roundout Creek, N. Y.	100	4.0 – 8.0	0.99	....	0.84	61.4
Catskill Creek, N. Y.	90	2.0 – 7.0	1.02	....	0.83	57.7
Reddie, N. C.	85	1.7 – 2.5	1.02	0.97	0.83	57.7
Cornell Canal	16	7.2 – 8.3	....	....	....	65.7
Cornell Canal	16	0.55– 1.9	....	....	....	54.3

From a consideration of these experimental results and the additional information obtained from the mathematical analysis of the distribution of velocities in a vertical, it is evident that, although the position of the thread of mean velocity is approximately at six-tenths the depth, its actual position in any vertical varies and will be fixed by the position of the thread of maximum velocity. Consequently then, careful consideration should be given to the choice of the relative depth to be used for each channel.

This relative depth may be obtained by first making point measurements and then graphically and by calculation determine the position; or it may be obtained by relying on one's judgment after making an inspection of the conditions existing at the measuring section. As a guide in making such an inspection and in properly evaluating it, the following conclusions drawn by E. C. Murphy[1] from the results of extensive studies made by the U. S. Geological Survey, are given.

[1] *U. S. Water Supply Paper* 95.

1. The distance of the thread of mean velocity varies from 0.5 depth for small depths to 0.75 depth for larger depths.

2. The distance of the thread of mean velocity below the surface increases with the ratio of depth to width. For shallow streams in general it varies from about 0.55 to 0.65 depth.

3. In broad shallow streams, from 3 to 12 in. in depth, and having sand or fine gravel bed, the thread of mean velocity is from 0.50 to 0.55 depth below the surface.

4. In broad streams, from 1 to 3 ft. in depth and having gravelly beds, the thread of mean velocity is from 0.55 to 0.60 of the depth below the surface. The single-point method of holding the center of the meter at 0.58 of the depth below the surface will give good results.

5. In ordinary streams, where the depth varies from about 1 to 6 ft., the thread of mean velocity is about 0.6 below the surface.

6. In the smaller streams (creeks), having a width of from 20 to 40 ft., the thread of mean velocity is farther below the surface than in a broader stream of the same depth.

7. A rough bed causes a change at the lower end of a vertical velocity curve, and tends to raise the point of mean velocity in the section so that a measurement at 0.6 depth would be too small.

8. A smooth bed tends to make the measurement at 0.6 depth too large.

9. The depth of a river has a marked effect on the point of mean velocity, this point in a shallow river approaching 0.55 and in a deeper one approaching 0.65 depth.

10. In general the error in using 0.60 depth method is well within the limit of accuracy in stream-gaging work.

33. Reduction Coefficients.—As has been explained, the mean velocity in a vertical can be obtained with a very close degree of accuracy, say 1 per cent, by making velocity observations at a number of points in the vertical and then from a curve drawn through the plotted points, determining the curve's area and dividing this area by the depth, the quotient being the mean velocity. It is apparent that to apply this method to a number of verticals across a stream would consume much time and it would be a great deal more desirable to have a method which would permit of the observing of a single velocity, and then, by the application of a coefficient to this velocity, obtain the mean velocity in the vertical. Such a coefficient is called a reduction coefficient and extensive observations have been carried on to determine its value for velocities measured in different parts of the vertical.

34. Reduction of Surface Velocity to Mean Velocity for the Cross-section.—The relation between the mean velocity in a cross-section and the velocity at any point in a cross-section is quite indeterminate and is certainly not such as to expect any close determination of the mean velocity in the cross-section by applying a coefficient to a velocity measured at any particular point.

For rough estimates, however, it is permissible to observe the central surface velocity in artificial channels, or the maximum surface in natural channels, and then apply a coefficient of 0.8 to this measured velocity in order to obtain the mean velocity for the cross-section. This method is probably no better than 10 per cent for accuracy and is justified only in such cases as for the purpose of reconnaisance, or during the period of flood conditions when close accuracy is not essential.

In the early days of water measurements in the canals at Lowell, Mass., this method was used by observing the velocity of surface floats in the middle of the stream and such coefficients as 0.847 and 0.814 were used to convert this float velocity to the mean velocity of the canal. Each canal had its own coefficient and it was questioned whether similar coefficients could be obtained for other canals where the conditions were not as favorable. It was, therefore, decided later to do away with this method.

Other experimenters endeavored to obtain values for such a coefficient and the results of their work show values of this reduction coefficient varying from 0.75 to 0.95 or nearly a 25 per cent difference. Two empirical formulas which have been used to obtain this coefficient are as follows:

Prony: $C = \frac{V}{V_{cs}} = \frac{V_{cs} + 7.78}{V_{cs} + 10.35}$, where V_{cs} is the central surface velocity. For ordinary ranges of surface velocity, this gives values of C from 0.77 to 0.87.

Von Wagner: $C = \frac{V}{V_{ms}} = 0.705 + .01V_{ms}$, where V_{ms} is the maximum surface velocity and has to be determined by several comparative observations. For ordinary ranges of velocities, C would vary from 0.71 to 0.81.

Perhaps Prony's formula is best adapted to relatively narrow, deep, vertically sided channels, while Von Wagner's formula applies best to a natural channel.

Bazin obtained the following formulas from studies of streams where the maximum velocity occurred in the surface.

$$V_{\max} - V_{mean} = 25.4\sqrt{R \sin \alpha}.$$
$$V_{mean} - V_b = 10.87\sqrt{R \sin \alpha}.$$
$$V_{\max} - V_b = 36.27\sqrt{R \sin \alpha}.$$

NOTE: V_{mean}, in this case, refers to the mean velocity over the whole section. R is the hydraulic mean depth and V_b is the bed velocity.

Bazin stated that regardless of the position of the thread of maximum velocity the formula

$$V_{\max} - V_b = 36.27\sqrt{R \sin \alpha}$$

holds true.

35. Relation of Maximum Velocity to Mean Velocity in a Vertical.—This relation of the maximum velocity to the mean velocity in a vertical is not constant, and, as a matter of fact, is the least constant of the relations of all other velocities in the vertical to the mean. However, for stream reconnaisance and flood measurements, this relation can be determined with sufficient closeness to be of convenience. Some of the coefficients obtained by experiments are as follows.

TABLE IV.—COEFFICIENTS TO REDUCE MAXIMUM VELOCITY TO MEAN VELOCITY

Name	Range in C	C
Ritter	....	0.85
Harlacher	0.79 to 0.91	0.84
U. S. Geological Survey	0.78 to 0.95	0.85
U. S. Geological Survey	0.76 to 0.91	0.85

In Table III a range there is shown from 0.80 to 0.95 with an average of 0.86.

Consequently, it seems fair to take 0.85 as the approximate average value of this coefficient, keeping in mind that it may be anywhere between 0.75 and 0.95, and therefore the probable accuracy of the velocity thus obtained may be no better than 10 per cent. As a guide in the choice of a coefficient, it may be observed that conditions which tend to depress the thread of maximum velocity (narrow, deep channels) will tend to increase the value of the reduction coefficient.

Because of this wide range in coefficient values, this method should only be used at times when other methods would be impracticable. Such times would be during floods when the current is swift and there is considerable floating debris. The meter when placed just beneath the surface is more easily held in position than if placed further down in the vertical and also, it can be quickly removed should occasion arise to require it.

36. Velocities at Mid-depth in a Vertical.—It has been claimed that the velocity at the mid-depth in a vertical has a greater constancy than the velocity at 0.6 depth and is, therefore, a more suitable point than the 0.6 depth at which to measure the velocity. This claim is not entirely valid in view of the results of experiments.

Humphreys and Abbot,[1] from their experiments on the Mississippi River, held that the ratio of the velocity at 0.5 depth to the mean velocity in a vertical was sensibly constant having a mean value of 0.98. Starling[2] found from other studies on the Mississippi River that this ratio was 0.96, ranging from 0.94 to 0.98; Cunningham,[3] from tests on the Ganges Canal, found the ratio to range from 0.92 to 1.08; from experiments by Ellis on the Connecticut River, and by other engineers the ratio was found to vary between 0.92 and 0.98.

Wheeler and Lynch[4] found from their experiments that the ratio of the mean velocity of the entire cross-section to the velocity at mid-depth was constant and was equal to 0.95.

Investigations by Allen and Griffin[5] of the comparative constancy of velocities at different depths in a vertical, using data obtained in a river, a rectangular canal, and an aqueduct, showed for the river and the canal that the least variation in the ratio of the mean velocity to the velocity at any depth occurred at mid-depth.

The Cornell University experiments[6] showed the following results:

[1] "Physics and Hydraulics of the Mississippi," 1851.

[2] *Trans. Am. Soc. Civ. Eng.*, vol. 34.

[3] *Min. Proc. Inst. Civ. Eng.*, vol. 71.

[4] See *Thesis* of Wheeler and Lynch, C. E. Dept., M. I. T., 1894.

[5] See *Thesis* of Allen and Griffin, C. E. Dept., M. I. T., 1908.

[6] *U. S. Water Survey Paper* 95.

TABLE V.—RATIO OF MEAN VELOCITY TO MID-DEPTH VELOCITY

Series	Maximum	Minimum	Mean
A	0.99	0.90	0.958
C	0.98	0.91	0.944
D	0.99	0.96	0.974

It would appear, therefore, that this ratio is not actually constant, but nevertheless, an average value of 0.95 can be used for obtaining approximate results.

37. Mathematical Demonstration of Position of Mean Velocity in a Vertical.—On the assumption that the vertical velocity curve is a parabola with a horizontal axis, several formulas may be derived for expressing the mean velocity in terms of two measured velocities. One such formula may be derived as follows (Fig. 18):

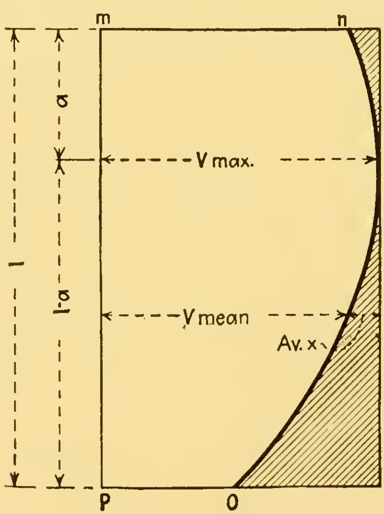


FIG. 18.

Let *nro* be a vertical velocity curve having a parabolic shape. Then the area *mnrop* will be proportional to the discharge of a section.

Proposition.—The area underneath the curve when divided by the depth will give the mean velocity, or the area outside the

curve (cross-hatched) when divided by the depth will give the average value of x which, when subtracted from the maximum velocity, will give the mean velocity.

The cross-hatched area will be

$$A = \tfrac{1}{3} \cdot a \cdot x_s + \tfrac{1}{3}(1 - a) \cdot x_b.$$

If the equation of the curve is

$$y^2 = 4px$$

then at the surface, where $y = a$, $x_s = \dfrac{a^2}{4p}$

and at the bed, where $y = (1 - a) x_b = \dfrac{(1 - a)^2}{4p}$.

Substituting in the above equation for the area

$$\begin{aligned} A &= \frac{1}{3} \cdot a \cdot \frac{a^2}{4p} + \frac{1}{3}(1 - a)\frac{(1 - a)^2}{4p} \\ &= \frac{a^3}{12p} + \frac{(1 - a)^3}{12p} \\ &= \frac{a^3}{12p} + \frac{1 - 3a + 3a^2 - a^3}{12p} \\ &= \frac{1}{12p}\left[1 - 3a + 3a^2\right]. \end{aligned} \tag{15}$$

Since the depth is equal to unity, this is also the average value of x.

If y_1 = the depth below the surface

$$x = \frac{(y_1 - a)^2}{4p}.$$

Therefore
$$\frac{(y_1 - a)^2}{4p} = \frac{1}{12p}\left[1 - 3a + 3a^2\right].$$

$$y_1 - a = \pm\sqrt{\tfrac{1}{3} - a + a^2}.$$

$$y_1 = a \pm \sqrt{\tfrac{1}{3} - a + a^2}, \tag{16}$$

which is the depth of the mean velocity below the water surface. This equation is similar in form to Eq. (14) previously derived.

From Eq. (16) it can be easily shown that in order to have the mean velocity occur at 0.6 depth, a, the relative depth of the maximum velocity, must be 0.15 in order to satisfy the equation.

38. Determination of Mean Velocity from Two Measured Velocities.—If y_1 = the relative depth of the upper point below the water surface and

y_2 = the relative depth of the lower point below the water surface,

then we may write

$$\text{Av. } x = \frac{\dfrac{(a - y_1)^2}{4p} + \dfrac{(y_2 - a)^2}{4p}}{2}$$

$$= \frac{2a^2 - 2y_1a + y_1^2 + y_2^2 - 2y_2a}{8p}.$$

The average x also $= \frac{1}{12p}[1 - 3a + 3a^2]$.

Equating the two values for x and collecting similar terms there results the equation,

$$y_1^2 + y_2^2 - 2a(y_1 + y_2) + 2a - \tfrac{2}{3} = 0.$$

With $y_1 = 0.21$ and $y_2 = 0.79$, the above equation is satisfied. These values, incidentally, cause the relative depth, a, of the maximum velocity to disappear from the equation, thereby showing that the relative depth of the maximum velocity will have no effect on the mean velocity obtained in this manner.

It is customary to use 0.2 and 0.8 as the points at which the velocity is measured and the error introduced is negligible.

In the observations mentioned in Art. 32, the ratio of the mean velocity in the vertical to the average of the two-tenths and eight-tenths was 1.001 for 476 curves and 1.005 for another set of observations, ranging from 0.97 to 1.026 in the first set and from 1.00 to 1.016 in the second set. Incidentally, this second set was obtained on shallow streams and shows how well adapted this method is for such streams.

Because this coefficient is so nearly unity, and does not vary more than 2 or 3 per cent from unity, this so-called "two and eight-tenths method" is considered to give the mean velocity quite accurately and should be used for general gaging work particularly where the location of the point of mean velocity is uncertain.

39. Top and Bottom Velocities.—The method of measuring the velocity just beneath the surface and near the bottom and using the average velocity as the mean velocity in the vertical has sometimes been used. This method is not desirable as may be observed from the theoretical curve and from actual measurements.

The equation of the curve has shown that the mean velocity is equal to the average of the velocities measured at 0.21 and 0.79 the depth only, and therefore if the mean velocity is taken as

being the average of the top and bottom velocities, the resulting value will be in error. The results of experiments mentioned in Article 32 show the variation to be from 0.96 to 1.31 having an average of 1.13 and 1.07.

The bed velocity was found by E. C. Murphy[1] to be of quite uncertain value. In three series of experiments, previously described, he obtained the following results:

TABLE VI.—SHOWING RELATION BETWEEN MEAN VELOCITY AND AVERAGE OF SURFACE AND BOTTOM VELOCITIES AS FOUND FROM CORNELL UNIVERSITY STANDARD WEIR AND FROM CURRENT METERS IN CORNELL EXPERIMENTAL CANAL

Series	Ratio of bed velocity to mean velocity			Ratio of difference between mean velocity and average of bed and surface velocity in per cent	
	Maximum	Minimum	Average	Maximum	Minimum
A	0.88	0.66	0.80	+27.5	+5.4
C	0.89	0.58	0.75	+30.6	−0.2
D	0.89	0.80	0.85	+ 6.6	−2.2

From these two tables it is evident that the bed velocity is very undesirable to include in any measurement of a discharge because of its variable nature.

40. Three-point Method of Determining Mean Velocity.—Three points have sometimes been used at which to measure velocities in order to obtain the mean velocity in the vertical. There is little to be gained by this method as shown by the theory and actual measurements.

Various groupings of points are used but more commonly these two: top, mid-depth, bottom, and two-tenths, six-tenths, eight-tenths. They are used in formulas in one of two ways, depending upon the line drawn through the points being assumed to be, (1) a broken line,

$$V_m = \frac{T + 2M + B}{4},$$

$$\text{or } V_m = \frac{V \text{ at } 2/10 \text{ depth} + 2V \text{ at } 6/10 \text{ depth} + V \text{ at } 8/10 \text{ depth}}{4},$$

[1] *U. S. Water Supply Paper* 95.

Or (2) a parabola,

$$V_m = \frac{T + 4M + B}{6}$$

$$\text{or} \quad V_m = \frac{V \text{ at } {}^2\!/_{10} \text{ depth} + 4V \text{ at } {}^6\!/_{10} \text{ depth} + V \text{ at } {}^8\!/_{10} \text{ depth}}{6}$$

The adverse criticism of surface and bed velocities made in earlier pages applies here and makes any formulas employing them suitable only for approximations. Also, using the velocity at six-tenths depth makes that formula less reliable than the two point method using two-tenths and eight-tenths depth which were reliable. If three points were to be used, the best three would be 0.15, 0.50, and 0.85 depth.

CHAPTER IV

STREAM GAGING STATIONS

41. Gaging Stations.—Points on a stream where measurements of flow are made and records of daily gage heights are kept for the purpose of obtaining the daily discharge, are known as gaging stations.

42. Types of Gaging Stations.—The measurement of the flow is made by either one of two methods. The method most generally used is the velocity-area method. As the name implies, the method comprises two separate procedures, namely, the measurement of the area of the channel cross-section and the determination of the corresponding velocity.

Inasmuch as the mean velocity for the entire section cannot be determined by a single measurement, the cross-section is divided into a number of vertical strips of suitable widths and the mean velocity obtained for each strip. The summation of the products of each partial area and its corresponding mean velocity is equal to the discharge past the whole section. Stations where this method is used are known as velocity-area stations.

The second method is the weir method and the stations where the discharge is obtained in this manner are known as weir stations. This method involves the measurement of the depth of water on the top of the weir and the use of this depth or head, as it is called, in some weir formula.

43. Station Rating Curve.—The daily discharge is obtained from the record of daily gage heights by means of a station rating curve. A station rating curve, or discharge curve, is a graph showing the relation between the stage and the discharge of a stream at a gaging station.

The curve is obtained by plotting discharge, measured at various stages, against the corresponding gage heights. In general, points thus obtained will plot so as to define a smooth curve and all subsequent measurements, when plotted, should fall on or very close to the curve.

The accurate determination of the shape of the curve is vital because the rating curve serves as a basis for all later deductions as to the rate of discharge of the stream at this point. Consequently, considerable study should be given to the data collected for use in the construction of the curve.

The method of applying the gage readings to the curve to obtain the corresponding discharges is based on the tacit assumption that a constant relation exists between the stage and the discharge as represented by the curve. Under conditions of constant flow, such a relation does exist at certain sections due to the constancy of the velocity and slope at any stage.

VELOCITY-AREA STATIONS

44. Gaging Section.—In the establishment of a velocity-area station, there are three sections in the channel which should be considered. Each section has a particular bearing on the accuracy of the results obtained at any station. These three sections are the gaging section, the control section, and the measuring section.

The gaging section is the place at which the gage heights are observed. It is at this place where the stage-discharge relation is determined. The site chosen for the location of the gaging section should be of such a character as to give reliable measurements and afford a permanent relation between the gage height and the discharge.

45. Selection of Site for Gaging Section.—Before deciding on a site for a gaging section, a thorough reconnaissance of the locality should be made to determine the one most likely to give consistent and accurate results. The necessary desiderata to keep in mind in making the selection are (*a*) suitable channel cross-section at all stages, (*b*) stable bed and banks, (*c*) freedom from backwater, (*d*) easily accessible location of gage, and (*e*) good control section.

The reconnaissance is best made at low stage. At this stage, there is an opportunity to examine the nature of the lining of the bed and banks and also to determine velocity at low stage. Neither of these can be accurately estimated at medium or high stages. Observations of the stream should be made at high stages also, in order to determine whether or not the stream overflows its banks and, if it does, to determine to what extent the flow is interfered with by trees and undergrowth.

On streams which freeze over during the winter months, a reconnaissance should also be made under ice-cover conditions.

46. Size and Shape of Channel.—The channel should be of sufficient capacity to contain the flow at high stages if high-water records are desired. If the stream does overflow its banks, the stage-discharge relation is uncertain because of the varying conditions of flow. For very high stages, accurate records may

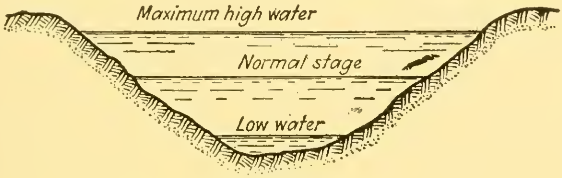


FIG. 19.—Cross section of stream to illustrate ideal shape of section.

not be necessary so that if the stream is contained within its channel until these very high stages are reached, the section may be entirely suitable. A channel with low banks, however, and subject to overflowing at medium stages is quite unsatisfactory.

The banks of the channel section should have gradual slopes. Ideally, these slopes should extend to the center of the stream

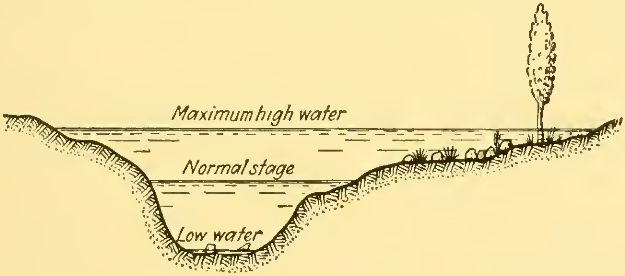


FIG. 20.—Cross-section of stream to illustrate undesirable shape of section.

forming a sort of rounded out V-shape (Fig. 19). This would afford good velocity even at low stage. However, the bed of the stream is often more or less flat, causing the flow to be rather sluggish at low stages. This sluggishness is not desirable for accurate results with the use of the rating curve.

A type of section which it is best to avoid is one having a deep portion through which the stream flows at low stages and a flat portion which is uncovered at low stages. With such a type there is a likelihood of aquatic vegetation springing up during

shallow water conditions which will seriously alter the stage-discharge relation (Fig. 20).

47. Stability of Bed and Banks.—The standard rating curve for the station will be constructed for a channel section of definite area and contour at any gage height. Any change in the area or contour at the control section for any gage height will therefore alter the stage-discharge relation at that gage height and make the rating curve inapplicable for obtaining the discharge. It is important, consequently, that the bed and banks be stable and no scouring or silting occur. The conditions under which scouring and silting occur were discussed in Chap. II.

48. Effect of Backwater.—Gaging sections should not be located where there is a possibility for water to back up due to ice jams, log jams, or other temporary obstructions formed below the section. Such action of the water causes the stage to rise without a corresponding increase in flow.

This effect is also produced at sections located too near the confluence of a stream and its tributary. A sudden increase in the flow of the tributary may produce a rise in the stage at, and a short distance above, the junction. The gage reading would consequently indicate an increase in flow in the main stream and greater discharges would be recorded than actually passed the section.

49. Accessibility of Gage.—The gage which is installed has to be read at all stages. Consequently, it should be easily reached by the observer at all stages. This point should be kept in mind when deciding on the best location for a gaging station.

50. Control Section.—To obtain a permanent stage-discharge relation at the gage the gaging section should be located above a control section. Such a section regulates or controls the elevation of the water surface upstream from it. In order for any given discharge to pass this section, the water surface upstream must have a definite slope and depth. Consequently, if a gage were installed at any point where the control is effective, the gage reading would always be the same for any given discharge. This ability to establish a definite relation between stage and discharge makes the control section a necessary requisite for any good gaging station.

51. Permanence of Control.—Once the rating curve has been constructed, it is desirable that the relation of stage and discharge remain constant. Any change in the character of the

channel at the control section will alter this relation and require the construction of a new curve. Consequently the control section should be permanent.

52. Sensitiveness of Control.—Since the control regulates the amount of rise and fall of the stage for changes in discharge, the magnitude of any change in stage corresponding to a given change in discharge will be determined by the shape and size of the control section. For example, a wide section would cause a smaller change in stage for any given change in discharge than would a narrow one. The ratio of the change in stage to the change in discharge is known as the sensitiveness of the control and the larger this ratio, the more sensitive is the control said to be.

53. Drowning of Control.—The effect of a control section is destroyed, if, for any reason, the control is drowned out by water being backed up above the section by some obstruction downstream. It may happen that, under low-water conditions, or

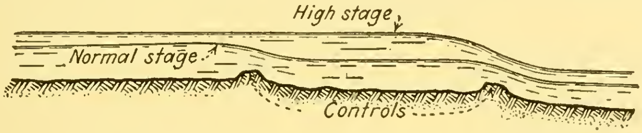


Fig. 21.—Illustrating the effect of the drowning of a normal control by another control at high stage.

even average or normal conditions, a control will be working perfectly, but as the water rises in the stream the backwater from a control section further down the stream will drown out the upper control and destroy the old stage-discharge relation and establish a new one (Fig. 21). Again, if a control is located just above the confluence of a tributary with the main stream, a sudden rise in the tributary may create a flood effect at its mouth and cause the stage to rise at the gage without any actual increase in the flow at the gage or the control section. This contingency should be considered when making a reconnaissance of the stream for possible controls, and also, due recognition should be given to the shift in the control in determining the stage-discharge relation.

54. Low-water Controls.—At low water, the control may be formed by an outcrop of bed rock, a gravel riffle, sand or gravel bar, or boulders. Sometimes the depositing of leaves and other debris will form a control. On some rivers in California, bars composed of tailings from placer mining have served as controls.

FIG. 22.—Control section on Winooski River at Montpelier, Vt. (*Courtesy, U. S. Geological Survey.*)

FIG. 23.—Control section, Otter Brook near Keene, N. H. (*Courtesy, U. S. Geological Survey.*)

Whatever the obstruction to flow is, it produces a break in the slope of the water surface. Above the control, the water surface is relatively flat, and below, it falls away sharply. As long as this change in slope is produced by the control, the control is effective.

Figure 22 shows a low-water control on the Winooski River, at Montpelier, Vt. This control is formed by an outcrop of bed rock. It is evident that this control cannot be effective at other than low stages. Figure 23 shows a control formed by small boulders. Figure 24 shows another control formed by

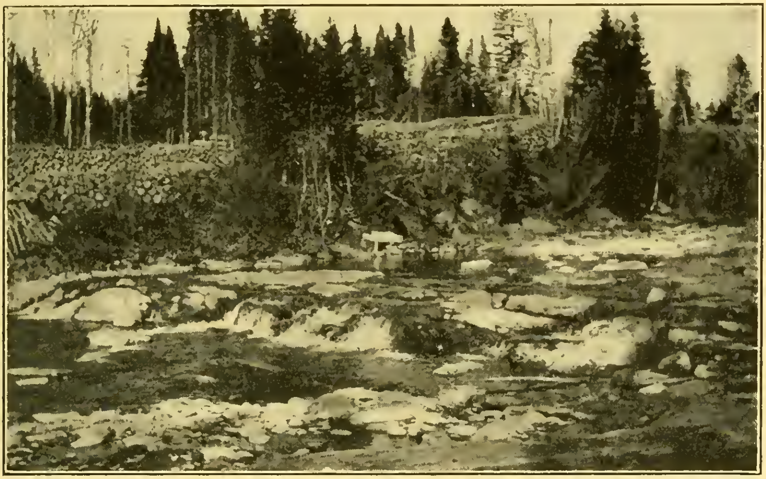


FIG. 24.—Control section, Connecticut River at First Connecticut Lake near Pittsburg, N. H. (*Courtesy, U. S. Geological Survey.*)

ledge rock and boulders extending across the stream. There are 3 ft. of fall immediately below the ledge which would make this control effective until high stages were reached.

55. High-water Controls.—At high stages, the low-water control is drowned out and the control section will be located downstream. This high-water control is generally permanent. It may be formed by steep rapids, large boulders, restricted passages, or the crest of dams. On streams having water power developments, it will often occur that a rise in stage will cause the backwater to extend from the dam several miles upstream and drown out the controls effective at lower stages. A long stretch of water of uniform slope and flowing in a well-defined channel with stable banks often serves as a control at high stages.

Figure 25 shows a control on the Souhegan River at Merrimack, N. H. This control is formed by a rock ledge at the head of some falls, and is permanent at all stages. Figure 26 shows a

Fig. 25.—Control section, Souhegan River at Merrimack, N. H. showing control free from ice-cover during winter months. (*Courtesy, U. S. Geological Survey.*)

Fig. 26.—Control section on the Pomperang River at Bennett's Bridge, Conn. River is at flood stage. (*Courtesy, U. S. Geological Survey.*)

control on the Pomperaug River at Bennett's Bridge, Conn. The control is formed partly by the rapids at this point and partly by the restricted section. Figure 27 shows another control also formed by a restricted passage.

56. Effect of Shifting Controls on the Station Rating Curve.— The station rating curve generally has the shape shown in

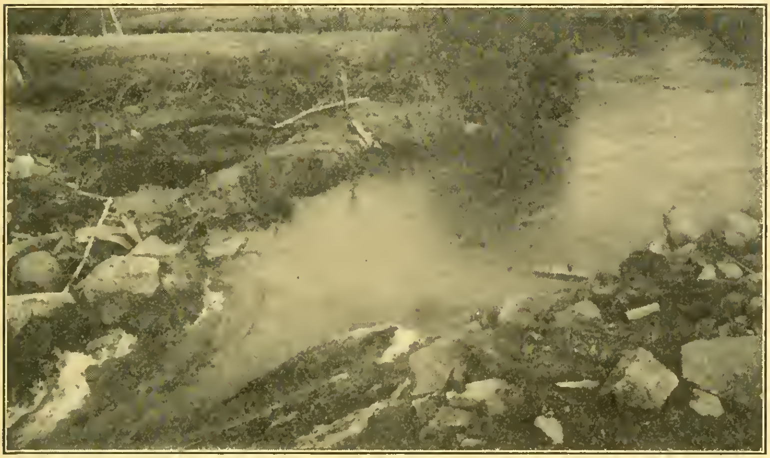


Fig. 27.—Control, Molly's Brook near Marshfield, Vt. (*Courtesy, U. S. Geological Survey.*)

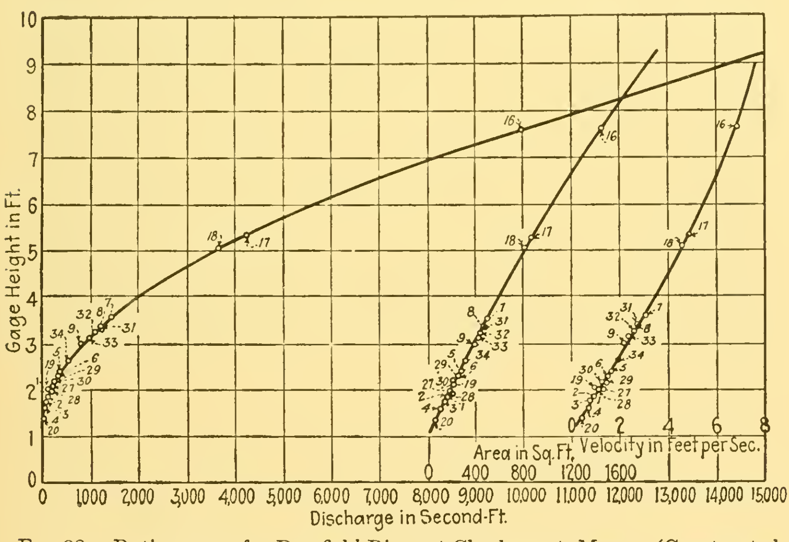


Fig. 28.—Rating curve for Deerfield River at Charlemont, Mass. (Constructed from curves in U. S. Water Supply Paper 415.)

Fig. 28. The slope of the curve is steep at the low stages and flattens out gradually as the stage increases. Such a curve may

be expected where the control is permanent and effective at all stages.

When the control is not effective at all stages the curve will not be smooth, but will have an abrupt change in its slope at the stage where the control shifts. Figure 29 shows a rating curve for a station where the control shifts. Up to about gage height, 4.2, the low-water control is effective. At this stage, the low-water control is drowned out and the stage is regulated by a second control located down stream. The shape of the upper portion of the curve is consequently determined by the high-water control.

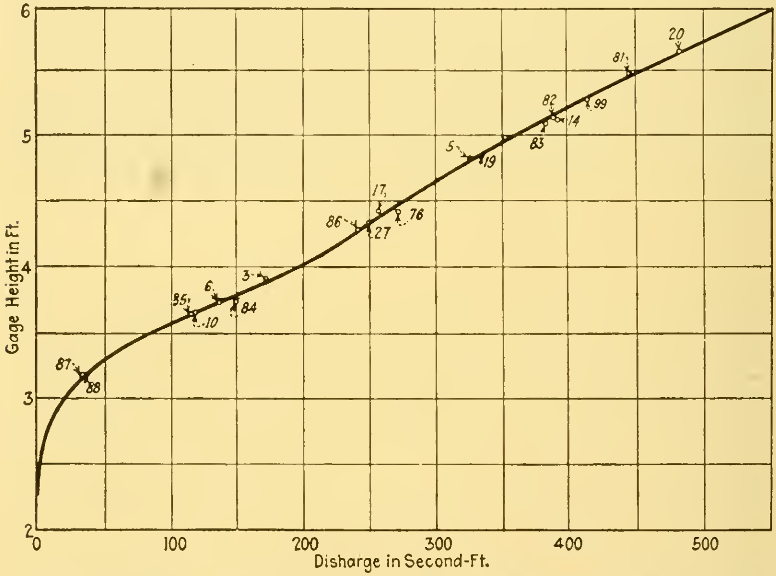


Fig. 29.—Rating curve for Millers River near Winchendon, Mass. (Constructed from curve furnished by U. S. Geological Survey, District Engineer's Office, Boston, Mass.)

57. Artificial Controls.—On some streams, it is impossible to find any satisfactory natural control. It will then be necessary to build an artificial control. Such controls are essentially weirs or dams, either submerged or having free overfall. They should be of such a shape as to offer but slight interference with the natural flow of the stream.

The point selected for the location should nave the general characteristics of the site of a natural control. There should be a free fall of the stream below the control, otherwise, for streams

containing silt, silting may occur downstream and produce a secondary control which will interfere with the effectiveness of the artificial control. It is desirable that the control should be high enough to be effective at all stages but the construction of such an artificial control may be impracticable because the natural slope of the stream is so flat that a high weir would have to be built if the control were not to be drowned at other than low stages.

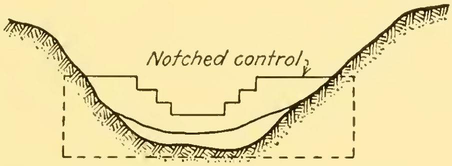


FIG. 30.—Illustrating artificial control of the notched type.

FIG. 31.—Artificial control on Logan River, Logan, Utah. (*From U. S. Water Supply Paper* 371.)

Artificial controls are made sensitive by making the section V-shaped or composed of a series of steps (Fig. 30).

The gage should be placed near the control in order to receive full benefit from it. All the water passing the gage should pass over the control. This means that there should be no leaks under or around the control. Flowing water has a very strong aptitude for seeping under or around the ends of such a structure

and, consequently, the control should be quite firm and securely fastened to bedrock either exposed or buried beneath the bed of the stream.

The control may be made of concrete, timber, or small boulders cemented together. Figure 31 illustrates the concrete type. Figure 32 shows an artificial control made of boulders cemented

FIG. 32.—Artificial control, Nubanusit Brook near, Peterboro, N. H. (*Courtesy, U. S. Geological Survey.*)

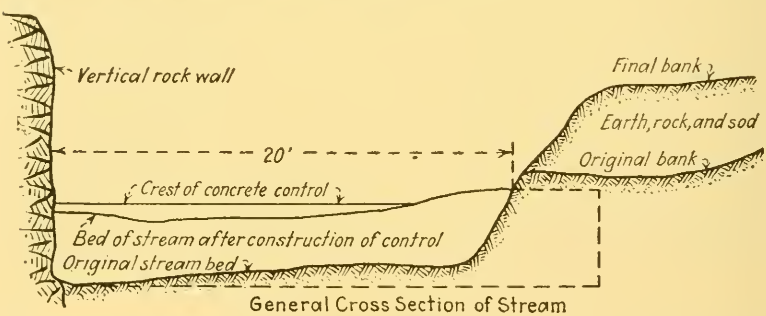


FIG. 33.—Artificial control. One type of artificial control recommended by the U. S. Geological Survey in U. S. Water Supply Paper 371.

together. Recommendations for the design of artificial controls are given in *U. S. Water Supply Paper* 371. Figure 33 is one arrangement of a control which is given in this paper.

58. The Measuring Section.—The point on the stream at which the discharge is measured is called the measuring section. This section may be the same as the gaging section or it may be

located elsewhere on the stream. In some cases the measuring section is located at a considerable distance from the gaging section. The only necessary provision regarding the relation of the two sections is that the same volume of water shall pass each section at the time of a measurement. This is necessary if the measured flow is to be compared with the gage height at the gaging station.

For float and slope measurements, more than one section have to be considered. These measurements require, respectively, a suitable course over which the passage of the floats may be timed, and one for which the slope of the water surface may be measured. For these measurements, the average of several cross-sections along the course is taken as a basis for determining the discharge.

Because of these various ways of measuring the velocity, velocity-area stations are classified as (*a*) current meter stations, (*b*) float stations, and (*c*) slope stations.

Since the measuring section serves a purpose different from that served by the gaging section, different considerations are given in its selection. The channel does not have to be permanent nor is a control section required. However, there are certain requirements to be met for a good measuring section and it may happen that different sections will need to be used at different stages in order that these requirements may be met.

59. Stream Alignment.—The measuring section should be located on a stretch of water which is sensibly straight. The flow over such a course will have the nearest uniform and stream-line motion possible. The presence of bends disturbs the regularity of flow and creates eddyings which affect the accuracy of measurements.

60. Shape of Section.—The shape of the section should be such as to provide suitable velocities for measurement at low stages. For this purpose a narrow V-shaped section would be better than a broad section with a flat bed. For the same discharge, at low stages, the broad shallow stream would have less depth of water and a more sluggish current than the V-shaped type with the result that measurements made at these stages would be less accurate for the former type of section.

The velocity at low stages should preferably be no lower than ½ ft. per second because current meters are generally unreliable when measuring velocities which are less than ½ ft. per second,

although with some meters velocities as low as $\frac{2}{10}$ ft. per second may be read accurately. As a rule, however, the effect of the friction of the bearings is too uncertain to permit of the measurements of velocities under $\frac{1}{2}$ ft. per second. The U. S. Geological Survey has a rule that does not accept velocity measurements where the velocity is below $\frac{1}{2}$ ft. per second in more than 15 per cent of the cross-section.

61. Smoothness of Lining.—Obstructions along the bed and banks introduce eddyings and other disturbances in the flow which cause inaccuracies in the measurement. The bed and banks should, therefore, be free from large stones protruding into the water, piers, ledges, outcropping trees, or vegetation, or any other objects which would interfere with smooth stream line flow.

WEIR STATIONS

62. Weirs.—Any obstruction placed in a stream so as to allow the water to flow over it is a weir. More particularly, however, a weir is any structure which is used to determine the discharge of a stream by measuring the depth of water on its top and the dimensions of the section through which the water passes.

The top of the weir is called the crest and the depth of water on the crest is called the head. The crest of the weir may extend entirely across the width of the channel of approach in which case it is known as a "suppressed weir." If the crest does not extend entirely across the width of the channel of approach, it is known as a "contracted weir." In the latter type, the width of the nappe or sheet of water flowing over the weir is less than the length of the crest due to the contractions of the nappe caused by the sides of the weir section. In the case of the suppressed weir the width of the nappe is the same as the length of the crest and the end contractions are said to be suppressed.

The weir section is the cross-section of the nappe taken in the plane of the weir crest. This section may have such shapes as that of a rectangle, a triangle, a segment of a circle, etc.

63. Sharp-crested Weirs.—A weir so designed that the nappe touches only the upstream edge of the crest is called a sharp-crested weir. This type of weir is known as the standard weir and is only used for the purpose of measuring water. If it is a contracted weir the sides of the weir section should be touched by the nappe only at the upstream edges.

The weir may be vertical or inclined but the vertical weir is the type more commonly used.

Most weirs are of the free overfall type, *i.e.*, the water level in the channel below the weir is lower than the crest of the weir. If the water level downstream from the weir is above the crest, the weir is known as a submerged weir. For this type, the head of the downstream water must be measured as well as that of the water upstream from the weir.

64. Broad-crested Weirs.—A weir over which the water flows so as to come in contact with the surface of the crest is known as a broad-crested weir. The most common example of a broad-crested weir is a dam. For such weirs, in addition to the head and crest length, such factors as the shape of the crest, the nature of the surface, thickness of the crest and the downstream slope must be measured in order to determine the discharge.

Where dams are used as weirs for the determination of discharge, it should be noted that all the flow of the stream may not pass over the dam. Some of the water may be diverted through canals or penstocks to turbines or other points of use. In such cases, the diverted flow will have to be measured by means of the turbines or by current meters, floats, etc.

65. Selection of Site for Weir.—The principal consideration to be given to the location of a weir is the height of the banks. It will be recognized that the building of a weir across a stream is going to raise the water level upstream from the weir. Therefore, the banks of the stream must be high enough to contain the water at all stages for which measurements are desired. The exact amount of rise in the water level for different volumes of flow can be calculated by assuming values for the elevation and length of the crest. From this calculation, the possibility of establishing a weir which can be used to measure the entire flow can be determined.

The foundation on which the weir is built should be strong enough to withstand the combined pressure of the water and the weight of the weir. Furthermore, the foundation must be impervious enough to prevent any water seeping under the weir. The effect of seeping is bad in two ways. It permits water to pass by the weir without being measured and it also creates an upward water pressure on the bottom of the weir which is liable to lift the weir from its foundation. Stream beds composed of

seamy rock or coarse gravel are apt to be rather pervious and should be avoided.

66. Choice of a Weir Station.—The specially built weir finds its greatest use on small streams where the channel is very rocky and current meter measurements are quite inaccurate. For wide streams, the difficulty of constructing a weir is great and consequently weirs are seldom built on such streams.

The use of coefficients derived from experimental studies places the limitation on the weir that it must conform to the experimental weir as regards shape of section, crest width and height, velocity of approach, and head. Any departure from these experimental conditions will introduce errors in the measured discharge.

In the case of dams being used to measure the discharge, inaccuracies of from 10 to 20 per cent may be obtained because of improper selection of coefficients. There is also the additional discharge through turbines, sluice gates, etc. which requires additional measurements. Consequently, the discharge records obtained at such a station may be greatly in error and the difficulty of collecting data may be so great that it would be preferable to establish a velocity-area station at some other point on the river.

CHAPTER V

GAGES

67. Definition.—A gage is some device which will enable the determination of the elevation of the water surface, or stage of a stream. This elevation, referred to some datum plane, is determined either directly by a graduated scale in contact with the water or indirectly by a graduated scale not in contact with the water but so placed that an index, which is moved by the rise and fall of the water surface, will indicate the stage on the scale.

68. Zero of the Gage.—The zero of the gage should be so located that the reading will never be negative. This is accomplished by placing the zero of the gage below the lowest known minimum stage. Where feasible, the zero reading should correspond to an empty channel, that is, the zero mark should be placed on a level with the deepest part of the section.

69. Bench Marks.—When the gage is set in place and the zero of the scale has been established, the gage should remain fixed. For one reason or another, however, it may be disturbed. For example, it may be struck by floating debris or ice, if located in the stream, or, due to ice action, it may be pried loose from its support, or the support itself may be dislodged, so that all the readings made after this movement of the gage has taken place, will be incomparable with those made previously. Consequently, the gage should be set with reference to at least one bench mark, and preferably more than one, in order that a check on the stability of the gage may be made from time to time. This certainty, at all times, of the correct elevation of the gage is absolutely necessary if the constancy of the relation of the discharge to the stage is to be maintained.

The intervals between times when the checks are made will depend upon the special conditions at the gage. In general, if the checks are made twice a year, say in the spring after the ice has gone out and the freshets carrying debris have subsided, and in the fall before the ice forms on the stream, the reliability of the gage will be determined.

70. Types.—Gages may be classified in two groups—the non-recording and the recording. The former may also be divided into two groups—the direct and the indirect.

The direct gage is commonly known as a staff gage because it consists of a staff on the face of which there is a graduated scale from which the position of the water surface can be read directly. With the indirect gage, the elevation of the water surface is transferred to a scale board by means of a rod or a chain and read there by means of an index.

Fig. 34.—Gage of U. S. Geological Survey on the Yukon River at Eagle, Alaska. (*From U. S. Water Supply Paper* 345.)

71. Vertical Staff Gage.—The vertical staff gage is the simplest type to construct and requires the least amount of care to maintain. It consists of a scale and a post to which the scale is fastened. The post should be securely fastened to some solid support or set upright in a concrete base placed in the bed of the stream.

The scale may be of wood or metal. If made of wood, it may be a pine board ½ to 1 in. thick and 4 to 6 in. wide, cut in 5- or 6-ft. sections. For the lower stages, it may be graduated in hundredths of a foot and at upper stages in tenths of a foot. Tbe face of the scale is preferably painted white and the gradua-

sions and numbers painted black. If the graduations are first cut into the board and then painted, the durability of the markings is enhanced.

The metallic sections are of cast iron or steel. The cast iron sections have the graduations and figures cast with the section itself. The steel sections are enameled and the graduations and figures are stamped on their face. When these sections are destroyed or the graduations obliterated they may be easily

FIG. 35.—Vertical staff gages, Piscataquis River near Foxcroft, Me. Gages are set to a common datum. This separation of the two sections of the gage illustrates one method of locating the gage so that it is accessible at all stages. (*Courtesy, U. S. Geological Survey.*)

replaced by similar sections. Figure 36 illustrates the different types recommended by the U. S. Geological Survey.

The foot and the tenth of a foot marks should be indicated to make the reading easier. If the tenths are not marked by number, it is quite easy to make an error in reading. It is not a bad idea to have the foot mark extend nearly across the face of the staff, the 0.5 half way across, and the 0.25 quarter of the way. Any device which will help to insure correct reading of the gage should be adopted, for all the care of measuring is undone by an incorrect reading of the gage. Figure 37 shows

a type of enameled gage most recently adopted by the U. S. Geological Survey. It will be noticed that the divisions are clearly indicated.

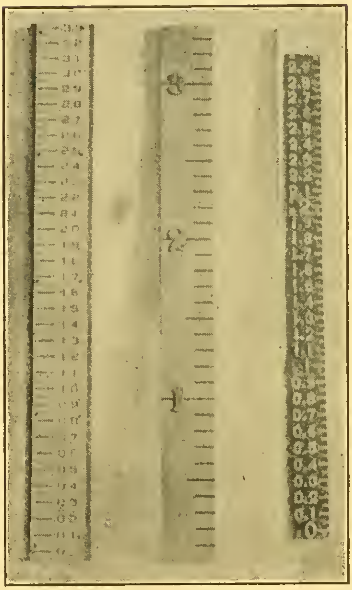


FIG. 36.—Types of staff gages used by the U. S. Geological Survey. (*From U. S. Water Supply Paper* 371.)

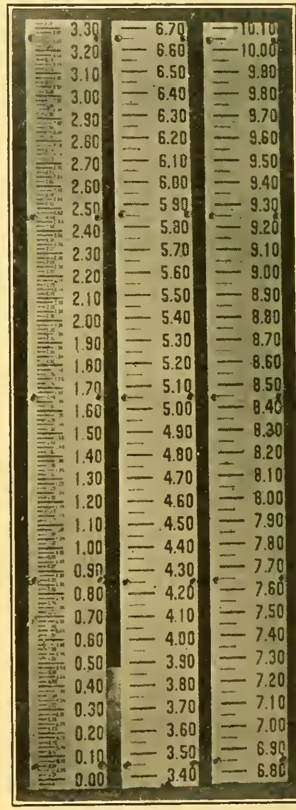


FIG. 37.—Metallic staff gage sections with enameled surfaces. Type used by U. S. Geological Survey.

72. Visibility.—The scale should be placed so as to be easily read. It is preferable to have the face of the gage squarely in front of the reader, to enable closer reading of the scale divisions. If the scale must face the stream, a platform extending out from the bank past the gage can be built. If this is done, the observer does not have to lean over the water in an uncomfortable position to make the reading, and look at the gage in a somewhat oblique position. The gage may be visible from a bridge. In this case, the figures should be large enough so that it is clear to

the observer just what the divisions are. When the gage is read in the winter, the ice should be cleared away from the gage so that open water exists at the gage.

73. Permanence.—The selection of a position for a gage should be made only after careful consideration of all factors affecting the readings of the gage. Shifting the gage about from time to time will give an opportunity for discrepancy in records because, for some reason, the gage might be read differently at different points, and the rating curve established for one position of the gage might not hold for another. The object, then, to which the gage is fastened should be as nearly permanent as possible. It may be attached to the pier of a bridge, the face of a retaining wall, a concrete post, a log, or other object. Any post erected to hold the gage should be sunk far enough into the ground so as not to be affected by any lifting effect of ice.

74. Stilling Box.—For close reading, it is desirable to have the gage located in a well or stilling box, placed in the bank of the

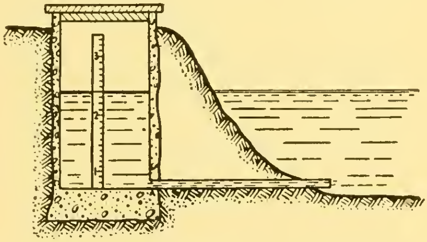


FIG. 38.—Illustrating the location and arrangement of a stilling box.

stream and connected with the stream by means of a pipe (Fig. 38). This insures a still surface of the water against the face of the gage and permits a very close reading. The well should be of ample size, say 3 by 4 ft., in plan and carried down far enough to permit low water readings. It should be connected with the stream by means of a 4-in. cast iron pipe laid on sills and supported at the stream end by a solid structure, such as a concrete pier. The stream end of the pipe should be screened to prevent any debris from entering the pipe and clogging it. In some installations, the box surrounding the gage is located in the stream and holes are bored in the sides to admit the water. In either installation, the elevation of the water surfaces in the well and in the stream should be compared from time to time. A common way to keep a check is to install a gage in the stream

and use that reading as a check on the closer one made with the gage in the well.

75. Inclined Staff Gages.—The staff gage is generally set in a vertical position, but where there there is no opportunity for a

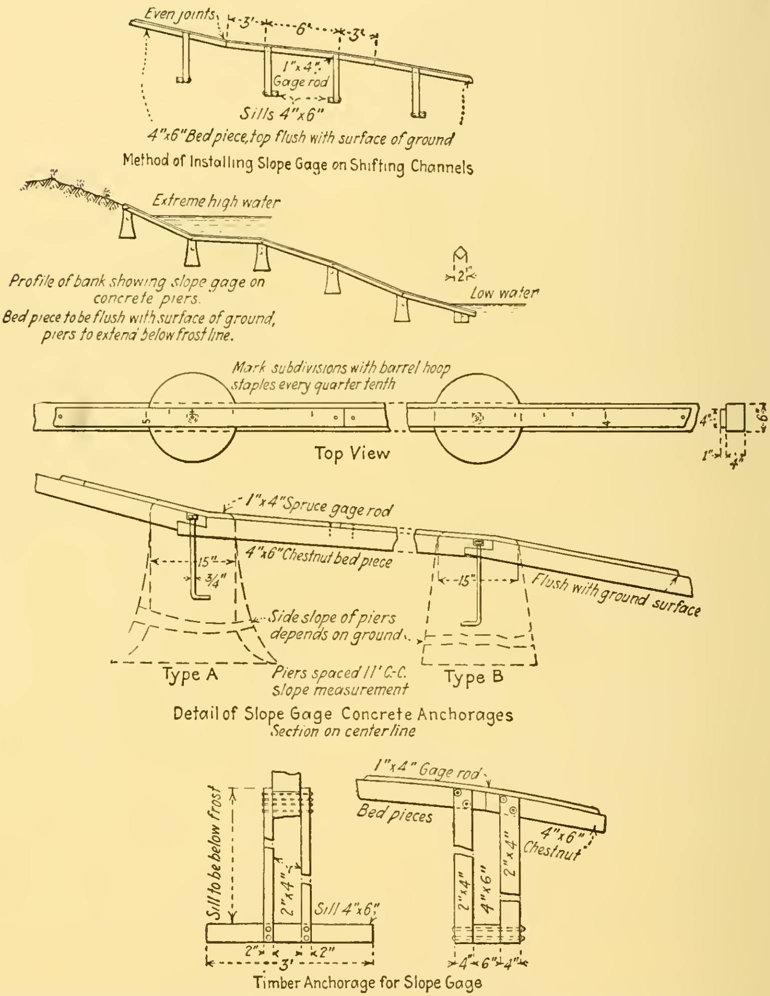


Fig. 39.—Plans for inclined staff or slope gage (constructed from Plate III, U. S. Water Supply Paper 371).

vertical gage, the gage may be placed on an incline. This is not as desirable an arrangement, for there is greater opportunity for the gage to shift its position due to the settling of its supports.

The scale board, made of 1 × 4-in. spruce is fastened to a 4 × 6-in. timber whose top face is laid flush with the surface of the ground. The timber is supported at short intervals by posts or concrete piers set firmly in the ground. Figure 39 illustrates the setting recommended by the U. S. Geological Survey.[1] The spacing of the divisions on the scale is made so as to read directly the vertical changes in water depth. When the slope has been determined upon, the corresponding sloping distance

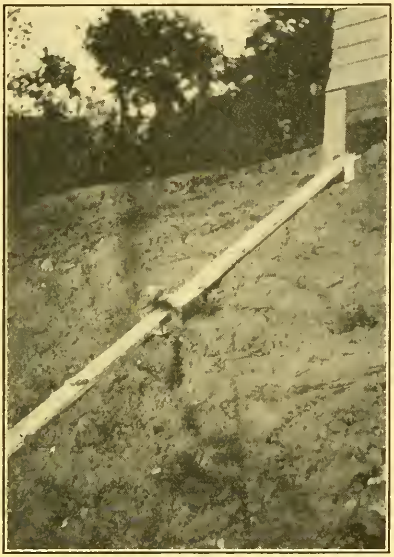


FIG. 40.—Inclined staff gage, Quinnabaug River at Jewett City, Conn. (*Courtesy, U. S. Geological Survey.*)

can be laid off on the scale, and the points marked either by cuts on the gage, by tacks, or by other means which may suggest themselves. Quite often the graduations are set on the gage by means of a level after the gage is set in place. Figures 40 and 41 show actual installations of slope gages.

76. Indirect Gage.—With the indirect gage, the rise and fall of the water surface is indicated by means of an index on a scale board placed above the water surface, the index being moved by some intermediate device such as a rod or a chain. Such a gage finds use where the staff gage could not be easily read or securely fixed. There are three general types of indirect gages, namely, (*a*) the float, (*b*) the weight, and (*c*) the hook gage. The first

[1] *U. S. Water Supply Paper* 371.

two are perhaps in more general use, while the hook gage is reserved for measurements where close readings are required.

77. Float Gage.—The principal parts of a float gage are the float, a rod or chain kept taut by a counterpoise and arranged with an index for the purpose of transferring the stage, and the

Fig. 41.—Inclined staff gage, Winooski River at Montpelier, Vt. (*Courtesy, U. S. Geological Survey.*)

scale for obtaining the stage reading. The index may be fixed and the scale appear on the rod. Such a gage should be located in a well or other enclosure to insure quiet water surface (Fig. 42). The float is described later under recording gages.

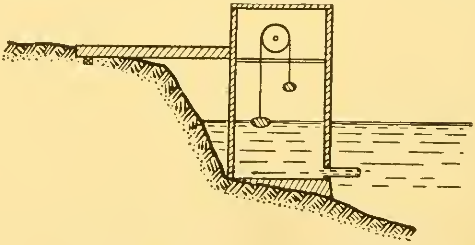


Fig. 42.—Illustrating arrangement of well for float gage.

78. Weight Gage.—With such a gage the direct measurement is made of the distances between the water surface and some point above the water surface. Such a gage is usually placed on a bridge or some overhanging object, and a reference mark

made on the bridge or other structure. The reading thus obtained has to be adjusted to read from the datum directly.

This type of gage is used a good deal by the United States Geological Survey[1] (Fig. 43), and consists of a scale board 10 ft. or more in length, which is contained, either wholly or in part, in a box supporting a pulley wheel, over which slides a chain, usually of the heavy sash type, to one end of which is attached the weight and to the other end the marker. The weight is

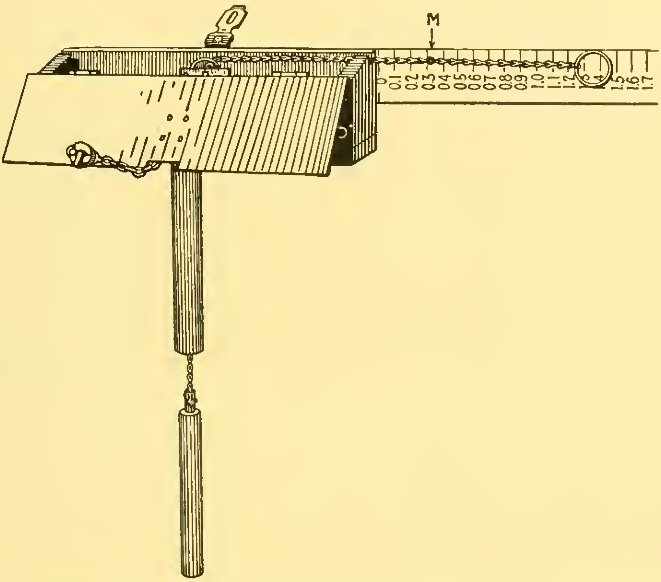


Fig. 43.—Weight or chain gage equipment. Type used by U. S. Geological Survey.

lowered until it comes in contact with the water surface, and the position of the marker is then read on the scale board, usually placed in a horizontal position. Incidentally, it has been the experience of the Survey that where the surface of the water is still, some difficulty is met in being certain when the weight was just in contact with it. Consequently, it is recommended that the water be moving when the weight strikes the surface.

Such a gage is not affected by ice and other floating debris and is reasonably stable but, on the other hand, errors may result from the stretch of the chain, wear of the pulley wheel, and deflec-

[1] *U. S. Water Supply Paper* 371.

tion of the structure. It is therefore necessary to make frequent checks to make sure that true readings are being obtained.

79. Hook Gage.—Hook gages are generally used at recording gage stations. The hook gage used by the U. S. Geological Survey (Fig. 45) is quite simple in its construction consisting of a graduated wooden rod placed in a vertical position and having on its lower end a hook which is bent through approximately 180 deg. The graduations on the rod are in an inverted position, that is, they increase in magnitude from top to bottom. The top graduation may be zero or any figure that may be required to conform to the gage datum and obtain the lowest water reading.

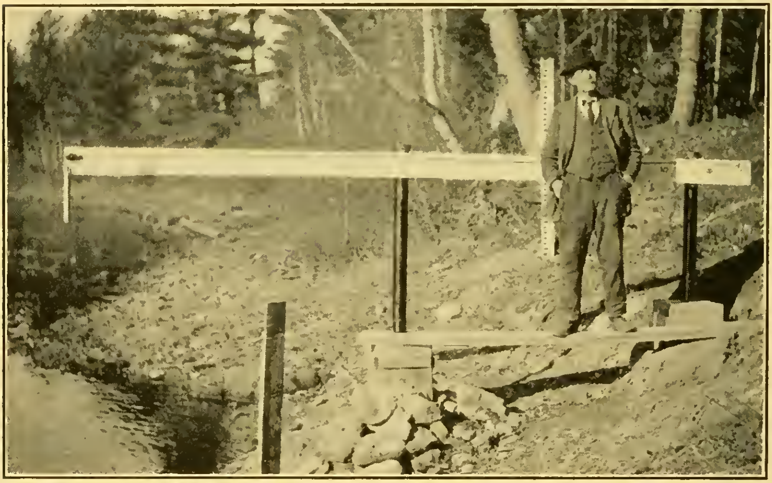


FIG. 44.—Weight gage installation, Passadumkeag Stream, Lowell, Me. (*Courtesy, U. S. Geological Survey.*)

The rod slides against a scale 1 ft. long and graduated to hundredths of a foot from zero upward. The zero of the scale should be put at a gage height elevation equal to the distance from the point of the hook to zero on the rod, or to the last figure on the rod plus its distance from zero.

The rod is free to move in a vertical direction, and in reading is lowered to such a position that the point of the hook is below the water surface. The rod and hook are then raised until the point causes a slight pimple to appear on the water surface, this pimple being due to capillary action of the water. When in this position, the rod is clamped and the reading is then made by means of the graduations on the rod and scale.

Another type of hook gage is that shown in Fig. 46.[1] This type is made entirely of metal. The rod slides in a fixed support provided with a vernier for reading to $1/1000$ of a foot. The support has a slot on either side of the rod for the purpose of fastening the gage to a post which is provided with two nails or screws spaced to correspond with the slots. The top of the slots are on a level with the zero of the vernier. The hook is made adjustable so that it can slide within the tube and allows for a movement of 12 in. independent of the gage.

FIG. 45.—Hook gage. Type used by the U. S. Geological Survey. (*From U. S. Water Supply Paper* 371.)

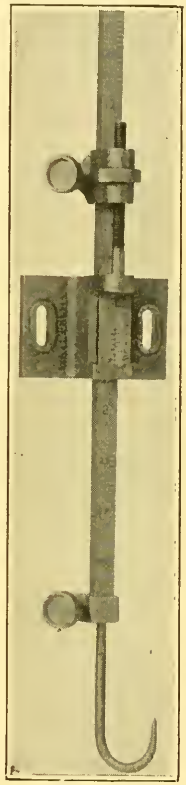


FIG. 46.—Metallic hook gage (*Courtesy, W. & L. E. Gurley Co., Troy, N. Y.*)

An advantage of this type of hook gage is its portability which has especial value where several gages are being maintained short distances apart on the same stream. At each point where the gage readings are to be obtained, stakes may be driven securely into the bank and nails driven into the post on which the gage may be placed to read the stage. In this way, but one gage is

[1] Manufactured by W. and L. E. Gurley, Troy, N. Y.

necessary and all that needs to be maintained is the post to support the gage. The two nails, or screws, or whatever is used for the gage to hang on should be set level and their position checked from time to time if the post is not secure, as it may be moved by cows, boatmen, floating debris, or by settling.

80. Recording Gage.—The recording gage keeps a continuous record of the stage of a stream, either by means of a graph, the coordinates of which are time and the elevation of the water surface, or by printing the gage reading at regular intervals by means of an intermittent printing device.

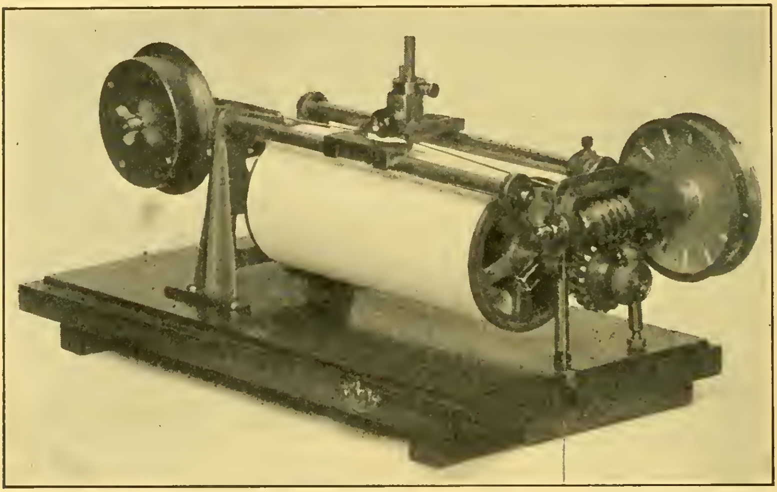


FIG. 47.—Gurley type water level recorder. (*Courtesy, W. & L. E. Gurley Co., Troy, N. Y.*)

The essential features of a recording gage are:

1. A float to rise and fall with the water surface.

2. A mechanism to transfer the vertical motion of the float to the record, either directly or at some reduced scale.

3. A sheet of paper to receive the record of the rise and fall of the float.

4. A clock connected to a revolving drum.

81. Float.—The float should have a sufficient water-line area to be sensitive enough to respond promptly to a change in the water surface elevation. This readiness to respond to slight changes in stage determines the accuracy of the gage, and, in turn, its value. Although the float is the simplest part of the

whole gage, its proper size and cross-sectional area are of great importance.

The recording mechanism is operated by the rise and fall of the float. In the Gurley type of recorder (Fig. 47) this action of the float is communicated to the recording mechanism by means of a perforated band which passes over the driving wheel on whose surface there are teeth which engage with the band, and in the Au recorder (Fig. 48) a cable is used instead of the band and the driving wheel is grooved to receive the cable. On the other end of the band or cable is a counterweight. The

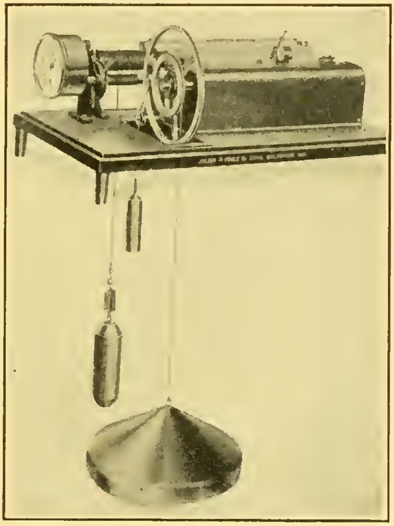


Fig. 48.—Au type water level recorder. (*Courtesy, Julian P. Friez & Sons, Baltimore, Md.*)

weight of the counterweight should be slightly greater than the sum of (*a*) the weight of the band or cable and (*b*) a weight sufficient to overcome the friction of the instrument when suspended from the counterweight side of the pulley. The weight of the float should be slightly greater than the sum of (*a*) the weights of the band or cable, (*b*) the counterweight, and (*c*) a weight sufficient to overcome the friction of the mechanism, when suspended on the float side of the pulley.

The mechanism is actuated by the buoyant effort of the float, the magnitude of which is determined by the water-line area of the float. This area will also determine the sensitiveness of the

recorder, and in order to get a maximum amount of displacement, it should be as large as possible.

When there is no movement of the water surface, the system of float, counterweight, and buoyant force of the water are in equilibrium, that is to say, the weight of the counterweight and line on its side of the pulley will be equal to the weight of the line on the float side of the pulley and that portion of the float above the water. If there were no friction in moving the mechanism, then, this state of equilibrium would always exist, and any movement of the water surface would result in a corresponding turn in the driving wheel. However, there is friction in the driving mechanism, and it requires power to move it. It must be accomplished by altering the equilibrium between the float and the counterweight by an amount equal to a weight sufficient to drive the recording mechanism. The instrument, therefore, will not move until an overbalancing force is created by the rise or fall of the water around the float to form a water column of cross-sectional area of the float which will be equal in weight to the force required on the rim of the driver wheel to operate the instrument.

Obviously, then, the larger the water-line area, the less will the rise or fall have to be to produce the overbalancing force, and the more sensitive the float will be. This difference in force obtained for a given rise or fall in the water column depending on the cross-sectional area is shown by the following table taken from the "Gurley Manual:"

Table VII.—Showing Comparative Buoyant Efforts of Floats of Various Diameters for Depth of Flotation of $\frac{1}{100}$ Ft.

Diameter of float, inches	Area of float, square inches	Height of column, inches	Volume of column, cubic inches	Force of weight of water column ounces
4	12.57	0.12	1.51	0.88
6	28.27	0.12	3.39	1.98
8	50.26	0.12	6.03	3.52
10	78.54	0.12	9.43	5.50
20	314.16	0.12	37.40	22.00

It can be seen, therefore, that the force varies directly with the square of the diameter.

82. Transfer Mechanism.—In the graphical water-stage register, the graph of the rise and fall of the water level is made by a pencil moving over the surface of the record sheet, placed around a cylindrical drum. The drum may be revolved by the clock and the pencil moved across the record sheet in a direction parallel with the axis of the drum by the movement of the float, as with the Stevens recorder (Fig. 49) and the Fuzee type of Au recorder or the drum may be revolved by the movement of the float and the pencil moved by a weight under clock control as with the Gurley recorder and the drum type of Au recorder.

The second method permits of an unlimited fluctuation in the water surface since the graph is made around the circumference of the drum and can be of indefinite length, each complete revolu-

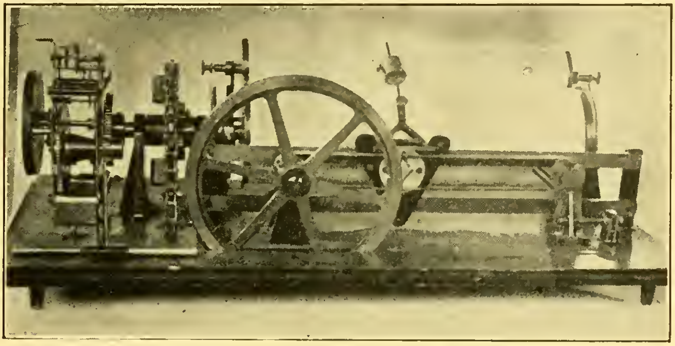


Fig. 49.—Stevens type water level recorder. (*Manufactured by Leupold & Voelpel, Portland, Oregon.*)

tion of the drum corresponding to a certain change in stage. The time covered by each record sheet will be limited by the width of the sheet, since the pencil will travel in one direction along the drum from one edge to the other. This arrangement is generally preferable, for no attention has to be paid to the range of the stage, and the sheets can be changed at fixed intervals of time, say, a day or a week.

In the printing type of register (Fig. 50) the stage is printed on a continuous strip of paper by means of two cylindrical type wheels having raised on their faces figures and divisions indicating, respectively, the number of feet and the hundredths of a foot. The foot marks on the Gurley register range from 0 to 36 and the hundredths of a foot from 0 to 100. There is also a third wheel arranged parallel with the other two which has

figures and divisions indicating the period of time from 1 to 12 hours divided into intervals of 15 min. This type wheel is controlled by a clock which is driven by a weight and will run continuously for a long time.

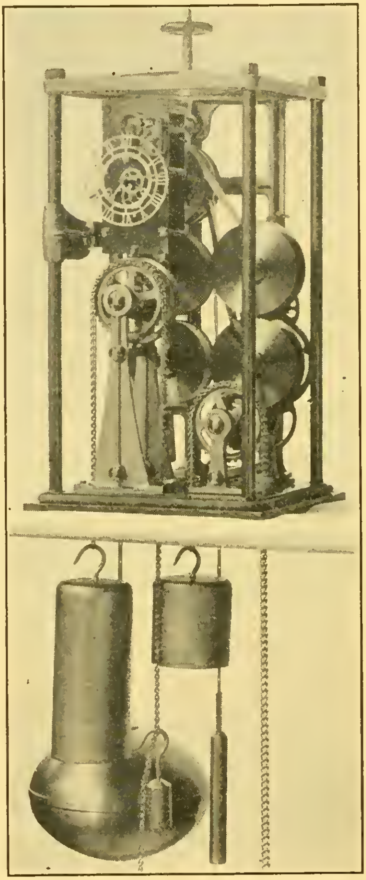


Fig. 50.—Water level printing register. (*Courtesy, W. & L. E. Gurley Co., Troy, N. Y.*)

83. Record Sheets.—The water-stage scale and the time scale should be accurately printed on the sheets used with graphic registers. The record sheet used with the printing type of register is a plain strip of paper 1 to 1½ in. wide, a roll of which is placed on one reel, passed over the type wheels, and wound up on the other reel. Between the record sheet and the type wheels there is a strip of carbon paper the same width as the record sheet,

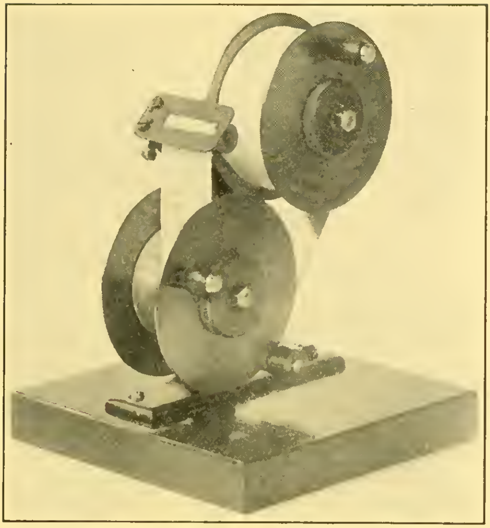


Fig. 51.—Tape reel. For use with water level printing register. (*Courtesy, W. & L. E. Gurley Co., Troy, N. Y.*)

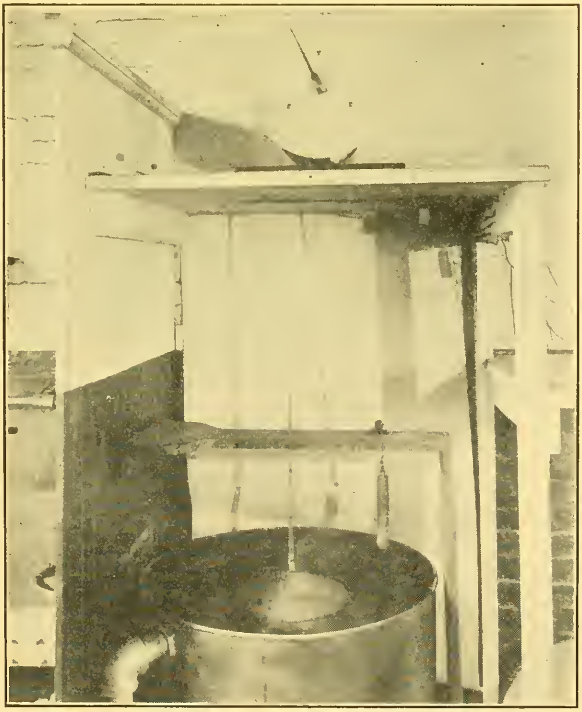


Fig. 52.—Water level indicator. Used in place of ordinary hook, chain and staff gages. (*Courtesy, W. & L. E. Gurley Co., Troy, N. Y.*)

which is carried in the same manner between the type wheels and the record sheet. A carbon impression is made on the record sheet when the printing hammer strikes. The printing is done by means of a hammer operated by a cam which allows the hammer to strike against the record sheet and the covering carbon sheet, thus printing the time and height of water stage on the paper record.

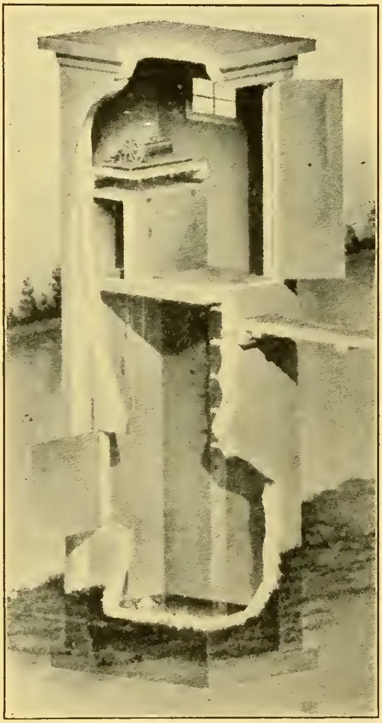


Fig. 53.—Typical water level recorder installation.

The printed type of record has an advantage over the graphic record, in that the relation of the time and the gage height is not affected by any change in the size of the paper caused by moisture.

84. Weight-driven Clock.—The clock is driven by weights. It should be durably built to withstand any hard usage. The clock should be compensated for temperature and its escapement should have perfect adjustment. A covering should be provided for the clock to keep out moisture and dust.

85. Essentials for Proper Installation of Recording Gages.—To secure the greatest accuracy in the use of the recording gage, much care should be given to its proper installation. If this is not done, the results of the most accurate gage are liable to be in error. Every recording gage installation requires the provision of certain features which are: (*a*) well, (*b*) intake, (*c*) shelter, and (*d*) auxiliary gage referred to permanent bench marks. Figure 53 illustrates the typical arrangement of the several features of a recording-gage installation.

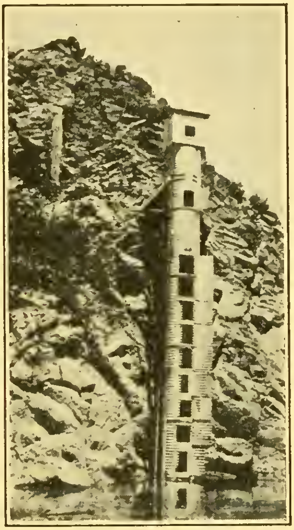


Fig. 54.—Well and shelter for water level recorder. Gila River, near San Carlos, Arizona. (*Courtesy, U. S. Geological Survey.*)

86. Well.—The well serves as a stilling box for the float. When conditions permit, it should be so located that the water may reach the well without the use of an intake pipe. It is found that the intake pipe is apt to become clogged by silt and the water does not flow freely between the river and the well, thereby introducing an error in the gage readings.

In climates where the water freezes during the winter months, it is desirable to locate the well in the shore at a distance from the river sufficient to remove the danger of being damaged by floating ice and other debris and to prevent the freezing of the water in the well. Several precautions may be taken to prevent the water in the well from freezing, such as locating the well far enough from the river so that the water level in the well is at least 2 ft. below the frost line, applying an oil cover to the water surface, or heating the well with an oil lamp or electric light bulb.[1]

The well should be made large enough to contain the float, the driving weight and counterpoise, and the hook or staff gage, and also have ample room for a man to go down into the well to inspect it. The United States Geological Survey recommends,[1] for a well that extends underground more than 8 ft., an inside cross-section of at least 12 sq. ft.

[1] *U. S. Water Supply Paper* 371.

The walls of the well may be built of tongue-and-grooved plank, creosoted or treated with some other wood preservative, brick, plain or reinforced concrete, or stone. Sometimes the well has been made of corrugated iron pipe where the installation was to be only temporary. Figure 54 shows such an installation on the Gila River, near San Carlos, Ariz.

The material used in the construction of the well will depend on its availability, the probable life of the installation, and the likelihood of floods or other conditions likely to destroy the well. Concrete, of course, is generally most durable but a good grade of lumber will provide a well that will have a reasonably long life.

The well should be provided with one or more doors in the sides if climatic conditions will allow it. These openings, placed at different elevations, are very desirable when reading the hook gage in deep wells because of the admission of light. They are useful also when cleaning out silt from the well. Where doors in the sides are impracticable, entrance to the well is made through a trap door in the floor of the gage house.

The well should also contain a permanent ladder extending to the bottom of the well to permit an examination of the float and intake pipe at low stages. The ladder should be fastened to the lining and be placed on the same side of the well that the door or doors are placed.

87. Intake.—The intake is for the purpose of conducting the water from the stream into the well and allowing the fluctuation of the water surface in the well to correspond to the change in the stage in the river. The intake should be level. If only one intake is used, it should be installed well below the lowest stage of the river in order that there may always be a registration of the water level on the gage. On some streams it is necessary to install intakes at two or more elevations to make certain of a connection with the stream at all times. This double installation is often required where silting is troublesome. The intakes are generally placed from 2 to 5 ft. apart.

The intake is best made of galvanized wrought iron, cast iron, or wood-stave pipe. The galvanized wrought iron and the wood-stave pipes should last at least 15 years unless the water is seriously polluted by chemicals. The cast-iron pipe will last from 40 to 50 years.

A 4-in. diameter pipe is ordinarily used, but in some cases 2- and 3-in. diameter pipes have been used. When there is considerable silting at the gage, the diameter may need to be as large as 10 or 12 in.

The river end of the intake should be securely anchored to prevent its being moved by floating ice and other debris.

A screen may be placed at the river end of the pipe to keep fish and other extraneous material from entering the pipe and clogging it. The screen will have to be inspected frequently to make sure that no obstruction has formed at that point. The pipe may be cleaned out by means of jointed rods pushed through from

FIG. 55.—Gage well and shelter, Pemigewasset River, Plymouth, N. H. (*Courtesy, U. S. Geological Survey.*)

the well end or by a chain drawn through the pipe. At the well end, there should be a valve to shut off the water and to reduce any wave action which may occur.

88. Shelter.—The shelter may be made of wood, corrugated steel, or concrete. Whatever type of house is to be used will depend principally upon conditions affecting the house. There should be ample room in the shelter for an observer to go into the house to inspect and adjust the recorder. There should be plenty of light in the shelter, which can be provided by means of a window. This window should be so placed as to throw light on the recorder so that the record may be changed and the instrument adjusted easily (Fig. 55).

89. Auxiliary Gage.—There should be two non-recording gages installed with each automatic recorder. One should be placed in the river in the same cross-section as the intake pipe, and the other, preferably a hook gage, placed in the well. The purpose of the hook gage is to aid in setting and checking the recorder and to indicate, by a comparison with the river gage, any interruption in the communication between the river and the well. Both gages should be referred to one or more permanent bench-marks.

90. Accuracy of Water Level Recorders.—For most purposes for which a water level recorder is used, the standard instruments now on the market are, in general, sufficiently accurate. However, when an unusual degree of accuracy is demanded, certain corrections may be applied to the indicated or recorded heights by which the true height can be obtained.

There are three sources of error for the counterpoised line which cause the indicated heights to deviate from the true heights. They are as follows:

1. The float has to perform a certain amount of work in moving the mechanism and therefore the indicated heights on a rising stage will differ from those on a falling stage by an amount which is proportional to the force required to operate the recording device. This may be termed the float lag.
2. A portion of the line shifts from one side of the pulley to the other and may be designed as the line shift.
3. The counterpoise may become submerged, thereby altering the balance of the system.

91. Computation of Errors in Readings of Water Level Recorders.—These sources of error were investigated by J. C. Stevens[1] who derived certain formulas to be used in making corrections for them, as follows:

Float Lag.—Let the following notations be used (Fig. 56):

W = weight of the float.
C = weight of counterpoise.
w = unit weight of water.
u = weight of one foot of line.
L = total length of line.
l = portion of the length of the line on the counterpoise side of the pulley.
D = float diameter.
A = area of float.

[1] *Trans. Am. Soc. Civ. Eng.*, vol. 83.

The lag of the float between a rising and a falling stage will be expressed by the formula

$$x_r - x_f = \frac{2F}{wA} \text{ or if the float is circular } \frac{8F}{w\pi D^2}$$

where x = the depth of flotation of the float with the counterpoise in the air. Subscripts r and f denote the rising or falling stage. F = the force required to operate the instrument.

To make this error as small as possible, the friction and mechanical work should be reduced to a minimum, and the float be made as large as practicable. The lag, however, can never be wholly eliminated.

F is equal to the friction in the bearings, gears, etc., of the pen or pencil on the paper, and the work of bending the line over the

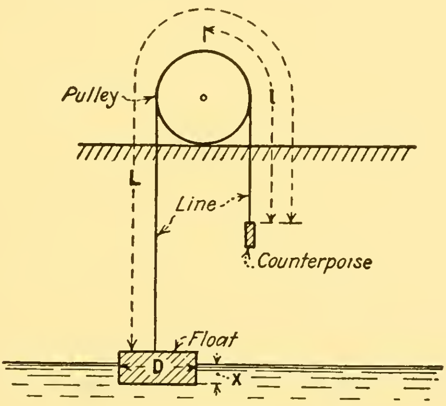


FIG. 56.

pulley. Its value will depend largely on the freedom of the instrument from dirt, oil, etc.

In practice, the indicator may be set to give the true height for a rising stage, then the quantity $\frac{8F}{w\pi D^2}$ can be subtracted from the reading at the same gage height on the falling stage to obtain the true height.

Example.—F = 1 ounce. D = 8 in.

then $x_r - x_f = 0.0057$ ft.

To insure good results within a reasonable limit of error, there should be used a large substantial float, a small counter-

poise, a light flexible line, and the instrument should be kept in good condition.

Line Shift.—This may be expressed by the formula

$$dx = -\frac{2u}{wA}(h_1 - h_0)$$

where u = the weight of line per foot.

h_0 = the elevation of liquid or gage height before change.

h_1 = the elevation of liquid or gage height after change.

Example.—1/16 in. g. i. sash cable. Weight = 0.0065 lb./ft.

$D = 8$ in.

$h_1 - h_0 = 10$ ft.

$dx = -0.006$ ft. This value is always negative.

Submergence of Counterpoise.—This will be expressed by the formula $x^1 - x = \frac{C}{S_c wA}$, always a positive value,

where

x^1 = depth of flotation of the float with counterpoise submerged.

S_c = specific gravity of the counterpoise.

Example.—1 1/4 lb. counterpoise 8-in. float.

$x^1 - x = 0.005$ ft. This compensates for the error of the line shift.

For a more complete discussion, reference may be made to Mr. Steven's paper.

92. Choice of Proper Type of Gage.—For well-chosen gaging stations, the discharge of a uniformly flowing, unregulated stream may be obtained from the rating curve by applying to it the gage readings made twice a day, usually in the morning and in the evening. Between these gage readings, however, sudden changes in stage may have occurred which will not be taken into account in the readings. It may be necessary, then, to install an automatic recording gage to take into account these variations. Whether or not a recording gage should be installed will depend on the conditions prevailing at the station which would make the two readings a day satisfactory. It might be noted that the two readings a day method may be preferable to the recording gage, if the records of the recording gage are improperly used.

Figures 57 and 58 show respectively the records of the hourly fluctuations of the Merrimack River at Franklin Junction, N. H., and the Quaboag River at West Brimfield, Mass. In both cases

the constantly changing stage requires more than one gage reading a day to determine the mean discharge for the day.

A study was made by the U. S. Geological Survey engineers in 1913, to determine and compare the error made in using two

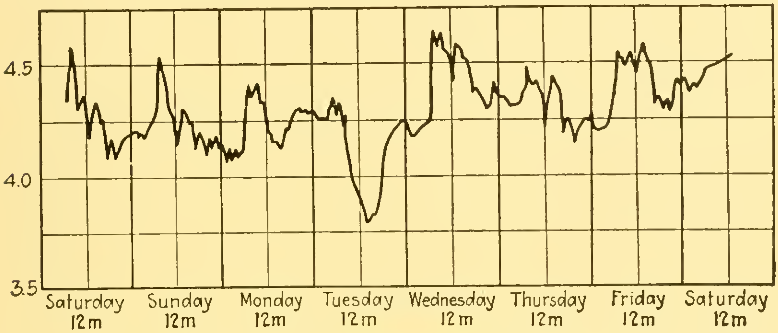


Fig. 57.—Continuous record of gage heights, Merrimack River at Franklin Junction, N. H.

gage heights a day, with errors due to incorrect methods of handling automatic gage records. They found that at a few stations better results would have been obtained with the

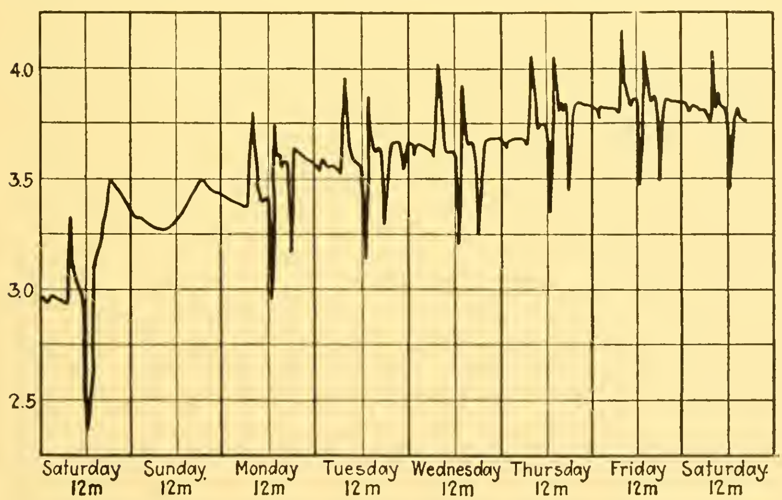


Fig. 58.—Continuous record of gage heights, Quaboag River at Brimfield, Mass.

two readings a day method than with continuous gage height records not properly used. For the two readings a day method, errors were generally compensating whereas the other method

caused cumulative errors. At other stations equipped with recording gages, it was found that the two readings a day method would have given results good enough for all practical purposes.

93. Conditions Requiring Automatic Gages.—C. H. Pierce in *U. S. Water Supply Paper* 375-*F*, discusses the conditions requiring the use of the automatic gages in obtaining records of stream flow and enumerates the conditions requiring automatic gages as follows:

1. The regulation of the stream by power developments.
2. Operation of canals and ditches delivering water for irrigation.
3. Fluctuations due to variations in runoff under natural conditions caused by (*a*) rain, and (*b*) melting ice and snow.
4. Inaccessibility of gaging station or lack of reliable observer.
5. Continuous record needed for legal purposes.
6. Human fallibility of most gage observers.

CHAPTER VI

CHANNEL AREA

94. Determination of Channel Area.—The cross-sectional area of a channel at any stage may be determined directly by soundings made at that stage or by measuring the water section at a low stage and developing the area at the higher stage by means of a profile of the banks extending from the low-water level up to the gage height in question.

In the latter method, the soundings, when referred to the elevation of the water surface, may be used to define the contour of the bed and banks below the water line. This portion of the contour can then be used to supplement the contour above the water line obtained by levelling. If this contour is plotted, the area enclosed by the submerged portion of the contour and the water surface, corresponding to any gage height, may be obtained by planimetering. This area will be that for the given gage height.

In this chapter, methods dealing with the determination of the area of the water section will be considered. In a later chapter, more will be said about the development of areas at higher stages and the method of constructing a standard area curve.

95. Spacings of Soundings.—The distance between soundings will depend, in general, upon (*a*) the width of the stream, (*b*) the evenness and shape of the bed, and (*c*) the accuracy desired. The U. S. Geological Survey has found that, for most streams, not less than fifteen or twenty soundings should be taken unless they are 1 ft. or less apart.

Soundings do not have to be spaced equally, the spacing often depending much upon the contour of the section. In the case where the bed appears to be so uneven that the profile of the section cannot be obtained closely by assuming the profile between the previously obtained soundings to be a straight line, intermediate depths must be measured. These intermediate points should be taken wherever there is an abrupt break in the profile. For other points, intermediate between soundings, the depths may be obtained by interpolation on the profile.

96. Closeness of Readings.—The closeness with which the depths should be read will be determined by the accuracy required, the nature of the material which lines the bed, and the depth. Soundings must be carefully made to be accurate. Where the bed is rocky, there is no necessity for reading closer than one-half of one tenth of a foot and generally the nearest tenth will give results consistent with the accuracy of any velocity measurements to be made later. However, it is usually true that greater errors in discharge measurements result from incorrect soundings than from incorrect velocity observations. The depth of the water will make some difference in the required closeness of the readings since the same error in a depth of 2 ft. and one of 20 ft. will differ in the percentage value. The percentage of error made in the measurement of an area will appear

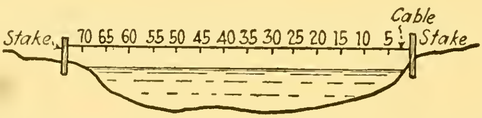


Fig. 59.—Illustrating method of locating soundings by cable stretched across stream.

in like magnitude in the error made in the discharge. This percentage of error is likely to be higher for shallow streams than for deep ones because of the greater relative effect on the depth of the sounding when the sounding rod rests on a small stone or other obstacle. For smooth, firm beds it may be reasonable to measure the depth to the nearest hundredth of a foot.

97. Location of Soundings.—In locating soundings for subsequent use in the calculation of the area or in the plotting of the profile, several methods may be employed, as follows:

1. *Direct measurement with the tape from the shore or some other reference point on shore* (Fig. 59). If the section is being taken in line with the edge of a bridge or other structure extending across the stream the points at which soundings are made may be located by taping from some reference point.

Where no structure is available the feasibility of taping will be determined by the width of the stream. Under these conditions, this method of taping is best adapted to streams of moderate width.

For narrow streams, the tape may be stretched across the stream and held taut while the soundings are made. For wider

streams, a rope or cable can be stretched across the stream and have foot divisions marked on it, or possibly have the tape tied to it.

For shallow streams, the observer can wade across the stream. For deeper streams, if no bridge or similar structure is available, a boat will be necessary. The boat can be fastened to a second cable stretched across the stream to prevent its moving with the current. The cable to which the boat is attached should not be used as the cable on which the soundings are located, for there would probably be a downstream sag in the cable which would result in the soundings being made out of the intended section. Where it is not feasible to have a supporting cable, the boat may be held steady by the oarsman.

2. *Stadia, on range, read from the shore* (Fig. 60). This method can be used with or without a cable. The cable serves

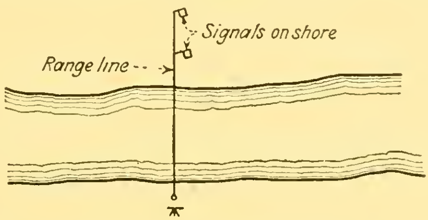


Fig. 60.—Illustrating method of locating soundings by means of range line and transit. Distances from transit measured by stadia.

the purpose of keeping the sounding boat on range. Without the cable, the sounding party must keep on range with points on shore. When a second cable is used, two men are necessary in the boat, one to make the sounding and the other to hold the stadia rod. With no cable to fasten to, it is necessary to have a third man to handle the boat and keep it stationary. The rodman should be near the one making the sounding in order that the appropriate distance to the point where the sounding is made may be read.

3. *Angles from base line on shore to sounding point* (Fig. 61). It is necessary in this method to establish a base line on shore and set up transits at either end of the line. With the vernier set at zero, along the base line, the angles from the base line to the boat can be measured simultaneously. The boat must keep on a range so that the soundings will be taken in the section. Signals can be arranged between the signal man in the boat and

the transit man on shore. Quite often, to synchronize the times of making soundings, two flags of different colors are used to help in identifying the different readings. It is a very good precaution to take to have a watch in each party agreeing as to time, and when a sounding is made or an angle read, the corresponding time noted in the notebook. This will often remove any doubt as to which readings go together. These methods of signalling and checking by watch are useful where the stream is quite wide and conversation between the boatman and the transit men is difficult.

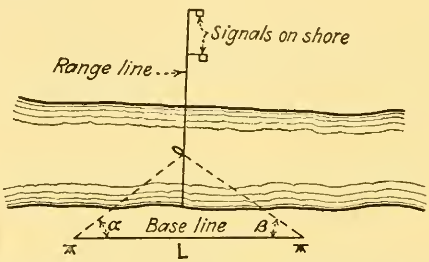


Fig. 61.—Illustrating method of locating soundings by means of range line and transits at either end of base line.

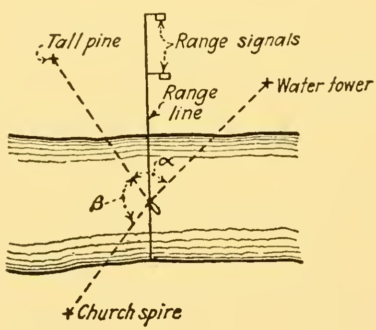


Fig. 62.—Illustrating method of locating soundings by sextant method.

4. *Sextants in the sounding boat* (Fig. 62). This method is well adapted to very wide streams. It requires that three objects be seen from the boat which can be located on the map. Two adjacent angles are then simultaneously read with two sextants. These observations are sufficient to locate the point. Full explanation of the sextant and its use may be found in various textbooks on surveying.[1]

[1] See Breed and Hosmer, The Principles and Practice of Surveying. Vol. II.

5. *Intersection of range lines* (Fig. 63). This method has its special use in large streams where soundings are taken periodically at the same point. By this method, the point of sounding can be easily found and the sounding taken. Ranges are established on the shore having intersections in the section whose area is being found. These ranges should be such that they can readily be picked up by the boatmen and the boat brought to the proper place in the stream.

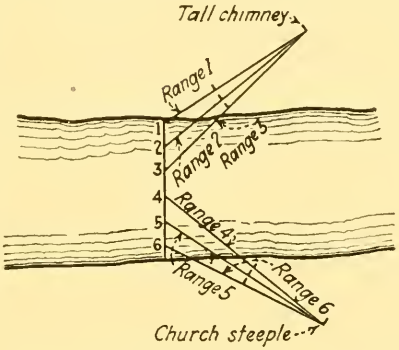


Fig. 63.—Illustrating method of locating soundings by means of several ranges.

98. Instruments Used in Sounding.—The depths are usually measured with either a rod or a lead line, depending on the depth and velocity of the water. Up to around 15 ft. and with low velocities, the rod can be used to advantage. For higher velocities, the dynamic force of the water against the rod is so great that it is quite difficult to hold the rod in a plumb position. For greater depths, and because of other reasons, which may make the rod undesirable, the depths are obtained by means of a line weighted down with some heavy weight of sufficient mass to insure the line hanging plumb. Such a line is commonly called a lead line and the weight is called the lead.

Fig. 64.—Sounding rod.

99. Sounding Rod.—The length of the rod should be suitable for the sounding to be made. It may be of one length from 10 to 15 ft., or it may be made in sections of 4- or 5-ft. lengths. These shorter lengths are more convenient for carrying. It should have sufficient thickness to be strong enough to stand the water pressure, but not too thick to make its handling difficult. For properly proportioned strength, it is sometimes made with a larger cross-section at the bottom than at the top. It is desirable that a wooden rod have a sharp edge in order to cause as little interference as possible with the flowing water. The rod is graduated in feet, tenths, and hundredths, the

zero being at the bottom, and if it is flattened on opposite sides, both sides have graduations. Care should be taken to see that the reading made is that of the true water surface and not that of the water running up the rod. The rod is generally made of wood, such as ash or hickory, which are tough, and is fitted at its lower end into an iron shoe provided with a flat base. The area of this base should be large enough to prevent the rod from sinking into the soft material when resting on the bed of the stream. In some cases, the bottom is provided with a cavity for the purpose of bringing up samples of the bed material (Fig. 64). For depths of 5 or 6 ft., a steel rod, $\frac{1}{2}$ or $\frac{3}{8}$ in. in diameter is sometimes used.

100. Sounding Line.—The lead line should be made of some strong, durable, and pliable material such as hemp. A hemp rope serves very satisfactorily as a line. Sometimes the line is made of piano wire or small chain. If the links of the chain are too large, however, the chain is not handled easily, especially when slipped over the edge of the boat.

If anything but metallic lines are used, they should be stretched before using. This is generally done by drawing the rope tightly between two posts, wetting it, and then allowing it to dry. After stretching the rope, it should be graduated, starting with zero at the bottom of the lead and marking every foot on the line by means of a piece of string or small piece of cloth drawn through the strands of the rope. The 5-ft. markings can be made distinctive as well as the 10-ft. markings. The line should be tested for length and any error should be recorded in the note-book and appropriate adjustment made of the depths as measured. The rope should be kept dry when not in use and soaked in water for an hour or so before being used. This will allow the rope to assume its tested length.

Where the water is swift it is necessary that the line be extra heavily weighted or sometimes what is called a "head line" may be used. This head line is for the purpose of keeping the sounding line in a vertical position.

Where the sounding is made from a structure at an elevation considerably above the water surface, a good method to use is to first lower the weight and line to the bottom of the stream, and, with the line taut, place a mark on the line at some fixed point on the structure; then raise the weight until it is just at the water surface and mark a second point on the line at the

fixed point on the structure. The distance between the two marks will be the depth of water in that vertical. Generally, it is not necessary to actually make marks on the line for the zero end of a metallic tape can be held with the fingers at the point on the tape when the weight is at the bottom and then the line and the tape can be run through the hand and the division on the tape observed when the weight is at the water surface.

101. Sounding Lead.—The lead or weight is made of iron or lead or other dense material. Sometimes window weights are used, or perhaps a weight of the same shape as a window weight but much larger. If the weight is of slender proportions it will slip through the water easily. The weight may have a cup-shaped cavity in its bottom to bring up the bed material as did the sounding rod shoe.

The weight of the lead required will depend upon the depth of the stream and its velocity. It should be sufficient to insure the sinking of the lead to the bottom and holding the line in a taut condition. Generally for streams of moderate depth 10-lb. leads are heavy enough. In some cases the lead will weigh as much as 25 lb.

It is well to have at least one extra set of lead line and lead so that should one be lost overboard the work can go on without interruption.

102. Allowance for Silting or Scour.—A section, when once measured, is supposed to remain the same but, as has been mentioned previously, some sections are subject to silting or scour so that the contour of the bed is constantly being changed and therefore soundings should be made at such sections whenever velocity measurements are made to insure having a correct cross-sectional area.

If soundings are not made when the velocity measurements are made, it is necessary that some approximation be made as to the depth at the time of the velocity measurement. Such an approximation calls for an assumption as to the rate of scour or silting.

If the scouring or silting is assumed to take place at a uniform rate between successive dates, and soundings have been made on these successive dates, the depths at any intermediate date can be easily obtained by a straight line interpolation.

This method may be illustrated as follows: if in the sketch (Fig. 65) d_1 and d_2 are measured depths made at a station on

different dates, and g_1 and g_2 the corresponding gage heights, at any intermediate date, the depth will be d_x and the gage height g_x. Also, let m be the number of days between g_1 and g_x, and n the number of days between g_1 and g_2. Then the depth d_x may be readily found from the geometrical relations of the figure.

103. Computation of Area from Soundings.—The total area of a cross-section may be obtained from the soundings by finding the small areas included between the soundings and then summing them up. The profile of the bed between soundings may be taken as straight or curving. The choice should be made after an examination of the actual profile. Part of the profile may best be considered as a series of straight lines, and another part

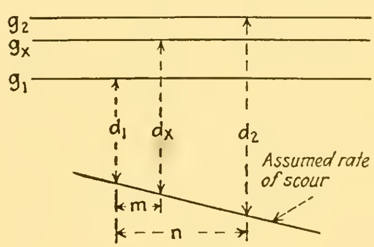


FIG. 65.

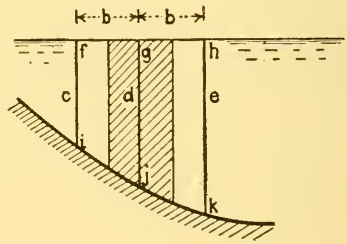


FIG. 66.

as a series of parabolic arcs. Methods for obtaining the area of a strip which may be used are as follows (Fig. 66):

1. The area of any strip may be planimetered.

2. For any strip such as *fgji*, regarded as a trapezoid, the area $= b\,\dfrac{c+d}{2}$.

3. For cross-hatched strip, regarding d as the mean depth, the area $= b \times d$.

4. For cross-hatched strip, regarding ij and jk as straight lines,

$$b \text{ area} = b\,\frac{c + 6d + e}{8}.$$

5. For a double strip, regarding each single strip as a trapezoid,

$$\text{area} = 2b\,\frac{c + 2d + e}{4}.$$

6. For a double strip, regarding ijk as a parabolic arc,

$$\text{area} = 2b\,\frac{c + 4d - e}{6}.$$

End areas may be estimated by any means suitable.

CHAPTER VII

FLOATS AND FLOAT MEASUREMENTS

104. Types of Floats.—There are three types of floats which are used in making float measurements, namely, (*a*) surface float, (*b*) double or subsurface float, and (*c*) tube or rod float.

105. Surface Floats.—Surface floats are designed to move with the same velocity as that of the water at the surface, and therefore should be made small and light in weight. The shape and weight should be such as to minimize the effects of any extraneous forces which act upon it to interfere with its travel. Such forces are the wind blowing against the exposed surface of the float and the surface ripples and eddies, the latter having an alternately retarding and accelerating effect. Figure 67 illustrates a suitable type of surface float. Even simpler types are sometimes used such as, corked bottles, oranges, blocks of wood, etc.

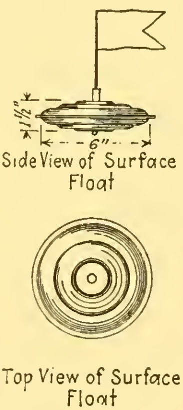


Fig. 67.—Views of a surface float.

The use of this type of float affords the quickest method of making a velocity measurement but, due to the action of the wind and the eddies in the water surface, the results obtained by it are liable to be considerably in error. Also, the float registers the velocity of but a few particles of water in the water surface, and then for only a short period of time, so that the relation between the observed velocity of the float and the mean velocity of the water is not at all fixed and any reduction coefficient based on this ratio may be quite inapplicable for accurate gaging. At best, this type of float is suitable only for measurements of streams at flood stage or for a rough reconnaissance of a river.

106. Sub-surface or Double Floats.—Figure 68 illustrates the make-up of a double float. It consists of a small surface float from which is suspended a weight slightly heavier than water.

The surface float serves to indicate the position of the float and to support the weight. The cord should be adjustable in order that the weight may be placed at any desired depth. The weight should offer large lateral resistance and should be circular in cross-section in order to offer equal resistance in all directions. It is made hollow in order to offer a small vertical resistance thereby lessening the effect of eddies and vertical currents. Ellis, in his Connecticut River experiments,[1] used a double float which consisted of a surface float of an ellipsoidal shape made of tin, 6 in. in diameter and 1½ in. thick having a cork in the top and a small flag inserted in the cork. The weight was connected to an eye in the bottom of the surface float by a cord whose diameter was 0.036 in. The subsurface float was a hollow annulus of tin 8½ in. high, 8½ in. outside diameter, and 7½ in. inside diameter. Two brass wires were soldered at right angles to each other across the bottom, and two others, in similar fashion, across the center. To the bottom wires, a 28-oz. weight was fastened and to the upper wires, the connecting cord.

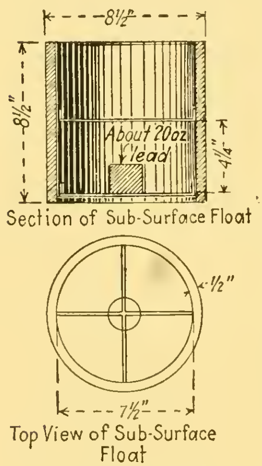


Fig. 68.—Views of a sub-surface float.

Ellis has stated that the following features are important in a good double float.

1. The lower float should offer large lateral resistance and this should be equal in all directions, on account of rotation.

2. It should offer small vertical resistance, so as not to be affected by eddies or downstream currents.

3. It should have sufficient stability of flotation to stand upright in the water.

4. It should have sufficient preponderance of weight to prevent floating upward in eddies and to keep the connecting cord vertical, but not such as unduly to increase the size of the surface float.

5. The surface float should be as light and as small as practicable and yet sustain the required weight, and its form should be such as to offer minimum resistance to the water and to the wind.

[1] *Report*, Chief Engineer U. S. A., 1878, Appendix *B*.

6. The connecting cord should be the smallest that is consistent with requisite strength.

This type of float is well adapted for deep rivers and in streams where there are floating weeds and grass. It is more reliable than the surface float because it is not subject to the wind and eddies to the extent that is true of the surface float. On the other hand, there are certain disadvantages to be found with the double float. It has the same disadvantage as the surface float in that it moves with a single pulsation of water and measures the velocity of only a few particles for a short length of time. In addition, the exact position of the subsurface float is uncertain. Its position is supposed to be indicated by the surface float and the

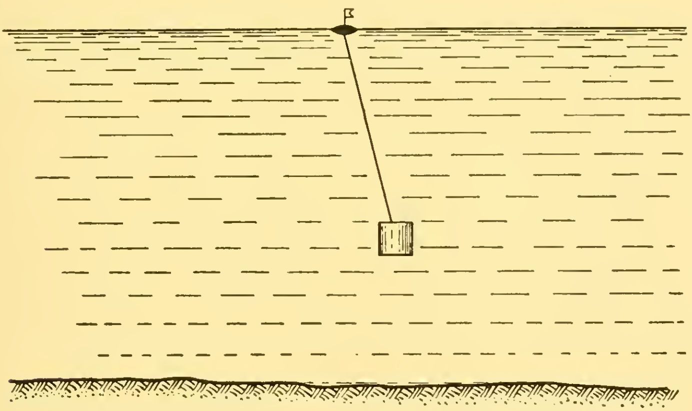


FIG. 69.—Showing relative positions of surface and sub-surface floats.

length of the cord, but due to the action of the wind on the surface float and the pressure of the water on the cord, the sub-surface float's position will vary and the surface float will not indicate its position (Fig. 69).

107. Tube or Rod Floats.—A tube float is a cylindrical tube made of wood or hollow metal 1 to 3 in. in diameter weighted at the bottom to keep the tube in a vertical position while floating (Fig. 70). The tube should be designed to float with a short length, perhaps 3 or 4 in. extending above the water surface in order to make visible to the observer the passage of the tube over the course. This projection also affords a place to grip the rod when immersing or withdrawing it from the water. The length of the tube should be such as to bring its bottom within a short

distance of the bed. If the tube extends too close to the bed, any bobbing due to improper immersion or to the eddying of the water will cause the tube to strike the bed and interfere with its passage.

Some metallic tubes are made of galvanized tin and some are made of brass. The brass is more durable and is much smoother, which is a twofold benefit in that the tube will slide easily through the hands when being immersed and will also slide through the water more easily.

The weight at the bottom of the tube may consist of shot poured into the tube, or it may consist of a lead weight inserted in the bottom of the cylinder. The weight is better because the shot is liable to be lost through the top which is generally sealed by a stopper. The better-made tubes will be free from seams of any kind and at the top on the portion extending above the surface of the water will be painted the depth of flotation of the tube.

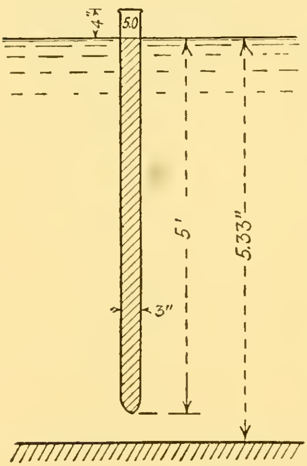


Fig. 70.—Tube float.

This type of float will give a better determination of the mean velocity of the water than the surface and double floats for it is acted upon by the water for nearly its whole depth.

Could the tube float so that it would extend the entire depth of the water, it would be acted upon by all the particles of water from the surface to the bed; but inasmuch as it is not possible to have the tube extend the entire depth, there is consequently a portion of the depth between the bottom of the tube and the bed of the channel where no effect of the water velocity is obtained. The velocity at this point is slow and if allowed to act on the tube would retard the velocity of the tube.

Since the velocity of a stream varies from the surface to the bed in accordance with a parabola having a horizontal axis, and the force acting on the tube at any point will be proportional to the square of the difference between the velocity of the tube and the water in contact with the tube, the ratio of the tube velocity to the mean velocity of the water will be dependent upon the ratio of the length of the tube to the depth of the water.

Cunningham[1] showed mathematically that V_t, the velocity of the tube, is equal to V_m, the mean velocity of the water, when d', the immersed length of the tube, is from 0.950 to 0.927 the depth of the water, the exact value depending on the location of the maximum velocity in the vertical.

James B. Francis conducted experiments in the canals at Lowell, Mass.,[2] to obtain a correction coefficient which would be applied to the tube velocity in order to obtain the mean velocity of the water, taking into account the effect of the difference in the length of the tube and the depth of the water. He found that the tube velocity was actually greater than the water velocity, using the mean velocity of a series of tube readings and the mean velocity of the water as determined from the discharge measurement of the water over a weir.

As a result of these experiments he derived the following coefficient:

$$C = \frac{V_m}{V_t} = 1 - 0.116\left[\sqrt{\frac{d - d'}{d}} - 0.1\right]$$

where C = coefficient.
V_m = the mean velocity of the water.
V_t = the measured velocity of the tube.
d = the actual depth of the water.
d' = the depth of immersion of the tube.

It can easily be shown that when the ratio of d' to $d = 0.99$ in the above formula, the tube velocity and the mean water velocity are the same. This expression for the coefficient was derived from experiments made in a rectangular channel about 9 ft. deep and with the ratio $\frac{d - d'}{d} = 0.12$. When applied to conditions other than the experimental conditions, slight errors are no doubt made. Parker[3] suggests that 0.2 is better than 0.116 for the constant in Francis reduction formula when applied to measurements in earthen channels and should have a still higher value for rougher linings.

Brown[4] made some tests on tubes varying in length from 7 to 11 ft., floating in water about 12 ft. deep, and found that by assuming Francis' coefficient to be correct for 5 per cent clearance

[1] Roorkee Hydraulic Experiments, 1881.

[2] J. B. Francis, Lowell Hydraulic Experiments.

[3] Parker, "Control of Water," Geo. Routledge and Sons, Ltd., London.

[4] *Wisconsin Engineering*, vol. VI.

below the tube, which was 0.986, the following coefficients would be obtained for greater clearances, based on the results of his experiments:

for 10 per cent clearance, $C = 0.969$.
for 20 per cent clearance, $C = 0.942$.
for 30 per cent clearance, $C = 0.919$.
for 35 per cent clearance, $C = 0.908$.

A series of experiments were made by two students at the Massachusetts Institute of Technology in 1909[1] where tubes of different lengths were run alternately with the standard long tubes. The experiments were made on two canals at Lowell, Mass. On one canal about 41 ft. wide, 8 to 9 ft. deep, and having velocities between 4 and 5 ft. per second, their coefficients agreed best with Francis' formula for clearances less than 35 per cent, in no case differing from the latter by more than 1 per cent. On the second canal, 48 ft. wide, 10 to 11 ft. deep, and having velocities between 2 and 3 ft. per second their coefficients agreed better with Brown's than with Francis', being practically the same in every instance.

In the Cornell experiments[2] the following results were obtained:

Table VIII.—Comparison of Tube and Weir Velocity Measurements at Cornell University

Average depth of channel, feet	Ratio: $\frac{\text{Weir measurement}}{\text{Mean of 5 rod floats per vertical}}$	
	$d' = 0.75d$	$d' = 0.90d$
9.3	0.989	1.003
8.3	0.955	0.973
7.5	0.962	0.980
6.3	0.960	0.971

108. Adaptability of Tube Measurements.—The tubes are best adapted for artificial channels of rectangular cross-section, because one length of the tube will suffice unless the canal be a feeder canal to some hydraulic turbines, in which case the depth is liable to fluctuate during a measurement and require tubes of various lengths. The bed of the channel should be smooth and free from obstructions. The tubes may be used in trape-

[1] See *Thesis* of Dort and Faulkner, C. E. Dept., M. I. T., 1909.
[2] *U. S. Water Supply Paper* 95.

zoidal shaped channels, but here, in addition to the requirement of tubes of widely different lengths, there is the possibility of a tube deviating in its run between cross-ranges and striking the sides, thereby spoiling the run.

For natural channels, the tube is not well adapted because of the changing shape of the cross-section along the course of the run and the possibility of striking boulders in the bed of the stream. For this reason the tube method is not often used for measuring the velocity of a river.

109. Details of Tube Measurements. 1. *Allowable Deflection.*—A straight stretch of water must be selected for the measurement, for the supposition is made that the tube will run parallel to the shore. For example, if the tube is 10.5 ft. out from the shore at the upper range, it should be 10.5 ft. out at the lower range. Due to eddies in the water, the tube may not hold to this intended course and will veer to one side or the other. The allowable amount of the deflection from the straight course will depend upon the length of the course. For example, for a course which is 100 ft. long, if the tube moves out of its course 1 ft. in the length of 100 ft., the actual course passed over will be approximately $\sqrt{100^2 + 1^2} = 100.005$ ft., which obviously may be considered as 100 ft. so that deflections are permissible and the limit can be calculated for any course and all runs where the deflection exceeds this limit should be thrown out.

2. *Actual Distance from the Shore Used for the Position of the Run.*—Where the tubes deflect in their course, the distance of the tube from the bank is generally taken as the mean of the upper and lower readings. Although this may not be strictly true, it is a very close and reasonable approximation to make.

3. *Length of Course.*—The necessary length of the course will be determined largely by the accuracy desired which is affected by the error in timing and the error in measuring. The shortest allowable length for a given degree of accuracy in the velocity to be measured can be obtained by a simple calculation.

For example, supposing that the allowable error in a velocity measurement is 2 per cent and in measuring the course there is a liability of one-tenth of a foot error in the measurement and an ordinary watch is to be used in the timing so that the error in reading the watch may be 1 sec. and further, that the maxi-

mum velocity of the water is taken at 5 ft./sec., the percentage error in length, L will be $\frac{0.1}{L} \times 100 = \frac{10}{L}$; the percentage error in time, T, will be $\frac{1 \times 100}{\frac{L}{5}} = \frac{500}{L}$. Since $V = \frac{L}{T}$ the resulting percentage of error in V may be obtained by $\sqrt{\frac{10^2 + 500^2}{L^2}} = \frac{501}{L}$. If this is not to exceed 2 per cent, then $\frac{501}{L} = 2$ and $L =$ 250.5 ft. This distance of 250.5 ft. will be the length of the run for which the probable error in the velocity will not be over 2 per cent.

Supposing instead of the course being laid out 250 ft. long, it were laid out only 100 ft., then the percentage of error in the velocity would be, approximately,

$$\frac{10}{100} + \frac{500}{100} = 5.1 \text{ per cent.}$$

By this means an estimate of the minimum length of course can be made.

4. *Ranges.*—The ranges should be placed perpendicular to the axis of the channel and have, on the upstream faces, graduated scales extending across the stream, the zero corresponding to the edge of the bank or a reference point on the shore. The divisions do not need to be nearer than 1 ft. and the tenths can be estimated by eye. The figures should appear opposite each division and be of suitable size to be easily read from the upper platform when the tubes are immersed. The points at which the tube passes the two ranges should be read and announced to the notekeeper.

5. *Structures for Immersing and Removing Tubes.*—The tubes are put into the water and taken out from platforms across the channel several feet upstream from the upper boom and downstream from the lower boom respectively. It is a good idea to have the platform adjustable so that it can be raised or lowered from a fixed platform to be a foot or so above the water surface.

The upper platform should be sufficently upstream from the upper range to allow the tube to attain its normal floating depth and velocity before crossing the range.

6. *Immersing the Tube.*—The tube is immersed in the water by a man stationed upon the platform. His best position is to

be kneeling on one knee. The tube should enter the water at an angle of 45 deg. with the vertical so that the tube is pointed well upstream. The tube is then rapidly slipped through the hands until the floating mark is in the water surface and then it is released. By pointing the bottom upstream and letting the tube slip rapidly through the hands, the tube will be in a vertical position when it is fully immersed, and will then float evenly and will not bob. If the tube is held after reaching a vertical position, the bottom of it will be carried downstream and when the top is released, it will swing forward resulting in a pendulum action so that the tube will accordingly start to bob. This, of course, interferes with the forward motion of the tube and, if too violent, will spoil the run.

7. *Timing.*—The time can be kept by means of a stop-watch operated by an observer on shore or in a boat if the tube cannot be seen from the shore. The watch is started as the tube passes the upper range and stopped at the lower. For ordinary courses the time taken to the nearest 0.2 second is close enough.

8. *Gage Reading.*—The gage should be observed for each run and any change noted in the notebook. Where the channel is feeding water to water wheels, there is apt to be frequent change due to the opening or shutting of the wheel gates so that close watch should be kept of the gage.

9. *Distance Apart of Runs.*—The intervals across the channels between the runs will depend upon the width of the channel. In general, the spacing in the central third of the stream can be greater than at the sides, for the velocities in the middle third are very nearly the same, but in the outer thirds the velocity slows up rapidly as the banks are reached.

As an example, in a canal 45 ft. wide, a run could be made as near to the wall as possible, at 0.5 ft. and every 0.5 foot for 2 ft., then every foot for 5 ft., then every 2 ft. for 10 ft., then every 3 ft. through the center and narrowing as the other side is reached making perhaps about thirty separate runs. However, the proper number of runs will be determined by the conditions.

10. *Calculation of Discharge.*—For the rectangular channel the depth will be constant. The velocity obtained by tube runs can be assumed to be the velocity at the middle of a strip and each velocity multiplied by its width. The sum of the products of the width and velocity divided by the width of the channel will give the weighted mean velocity of the channel. When

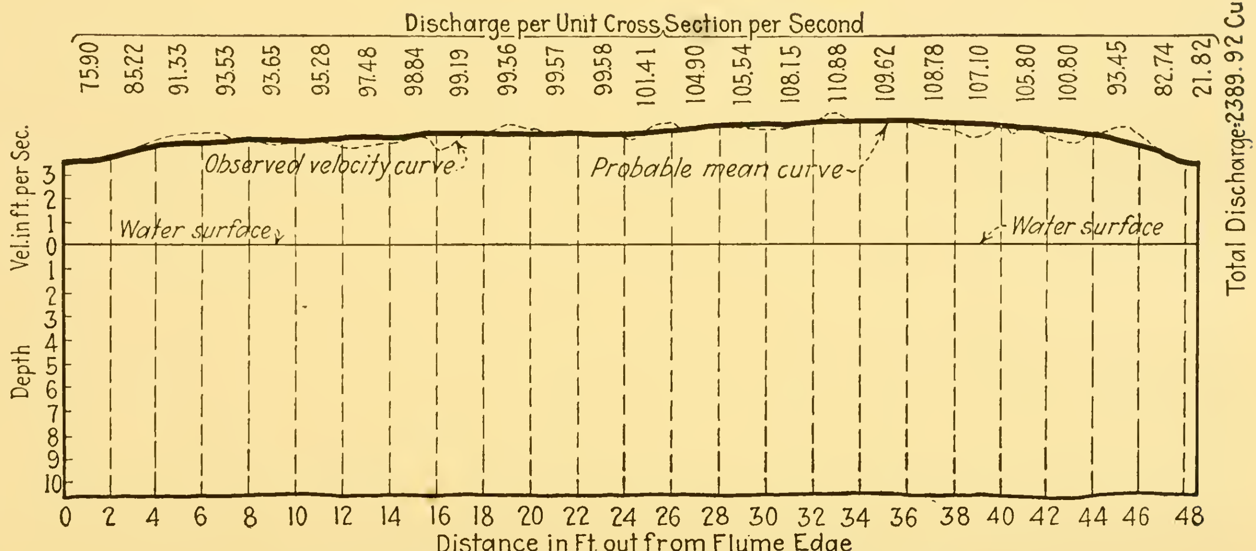


FIG. 71.—Flume measurements in Merrimack Canal, Lowell, Mass.

Length of run = 80 ft. Submerged length of tube = 9.5 ft. Average gage height = 10.55 ft. = average depth of water in flume. Watch correction negligible. Wetted cross-section = 512.7 sq. ft. Mean velocity corrected by Francis method = 4.65 ft. per sec. Computed discharge = 2384 cu. ft. per sec.

multiplied by the area the discharge will be obtained. Figure 71 is a plot of the results of a float measurement.

11. *Data to Be Obtained.*—The following observations should be made for each run.

1. Gage reading.
2. Depth of water.
3. Depth of immersion of the tube.
4. Distance out from bank at upper range.
5. Distance out from bank at lower range.
6. Time of run between ranges.
7. Distance of run between ranges.

110. Advantages of Floats.—The following advantages of the floats are to be noted:

1. They interfere but slightly with the natural velocity of the water, even when small.
2. They measure the velocity directly.
3. Adapted for use in streams of any size or velocity.
4. Rarely affected by silt or weeds.
5. Measure the velocity normal to the transverse section.
6. Inexpensive and easily repaired.

111. Disadvantages of Floats.—There are also certain disadvantages which must be balanced against the advantages. The following are some of the more serious disadvantages.

1. Difficulty in controlling their course.
2. They measure the velocity of a small number of particles of water which may be either maximum velocity or minimum velocity due to pulsation.
3. It is impossible to run the floats close to the bottom if it is at all irregular.
4. The size of the party must necessarily be large.
5. Several cross-sections are necessary to determine the average section.

CHAPTER VIII

CURRENT METERS

112. Types.—A current meter is an instrument which, when placed in a moving stream of water, will indicate the velocity of flow. It differs fundamentally from a float in that a float moves with the water and indicates the velocity directly, whereas the meter remains stationary and is operated by the velocity or pressure of the flowing water.

There are two classes of current meters, one depending directly upon the velocity of the water for operation, and the other upon the pressure of the water. The meters with the revolving elements are of the former class and the Pitot tube and various types of pressure plates, of the latter.

113. Development of Revolving Current Meter.—The first revolving current meter was that of Borda and Dubuat, which was used to obtain surface velocities. It was essentially a float wheel and suitable for use only at the surface of a stream. In 1790, Woltmann adapted this wheel for use below the water surface. His meter consisted of a pair of flat-shaped blades and carried an endless screw on its axis which operated a train of wheels by means of gears. The revolutions of these wheels were registered on dials. The meter was connected to a rod along which it could slide, the rod being firmly driven into the bed of the stream. The gears were engaged and disengaged from the screw by means of a string attached to the mechanism. Due to the ability of dirt to get into the gearing and the necessity of lifting the meter out of the water to read the dials, its use was not very extensive. In 1847, Baumgarten designed a meter with helicoidal shaped blades and increased the number of blades to four. Other experimenters endeavored to improve the meter by building it so as to lessen the friction in the screw and gearing. Changes were also made in the location of the dials so that it would not be necessary to lift the meter out of the water in order to read them.

D. F. Henry, finally applied an electric recording device to the meter. This was, perhaps, the greatest advance that had been made in the development of the meter as this method of indicating the revolutions of the wheel did away with the friction of the train of recording wheels. Other names such as Price, Haskell, Ellis, Fteley, and Amsler are associated with the development of the revolving meter and some of the meters which have been developed are described in subsequent pages.

114. Two Classes of Revolving Current Meters.—The revolving current meters which are in use at the present time may be divided into two classes, differing from each other in three respects, (*a*) the position of the axis of rotation, (*b*) the shape of the wheel, and (*c*) the action of the water on the wheel.

The first class, of which the Price and Ellis meters are the sole examples, has a vertical axis, a cup-shaped wheel, and its rotation is due to the difference in pressures on the opposite sides of the cups. The second class, of which the Fteley-Stearns and Haskell of America, Amsler and Stoppani of Switzerland, Richard of France, and Ott of Bavaria are examples, has a horizontal axis, helicoidal shaped vanes, and its rotation is due to the direct pressure of the water on the wheel.

115. Price Meter.—The Price meter is at present made in two patterns, the electric (Fig. 72) and the acoustic (Fig. 73). For both patterns the meter consists of a ⊐-shaped yoke carrying the wheel, made up of six conical-shaped cups, which rotates counterclockwise on a vertical axis. This axis, or shaft, is made of one piece and rests and turns on a hardened steel cone bearing, being supported laterally at the top. The pivot or cone bearing is made with an angle of 90 deg. and slips into a reamed hole in the yoke. The cone bearing is carried by a screw bushing which screws into a hub and onto the threaded end of the shaft. The cup wheel is clamped to a hollow hub through which the shaft slides to a shoulder located on the inside of the upper part of the hub.

The upper part of the shaft passes into a contact box where the electrical contacts are made. In the latest type, there is but one head or contact chamber, and contacts for indicating each revolution and each fifth revolution of the cup wheel are contained in this chamber. Two binding posts on the outside of the contact chamber are provided to change from single to penta-indicating. The positive lead is connected to these terminals

while the negative lead is connected to the frame of the meter, generally on the screw which fastens the weight hanger to the frame.

The signals are generally indicated by means of a buzzer or by a telephone receiver arranged in the electric circuit. The electric current is supplied by one or more dry cells.

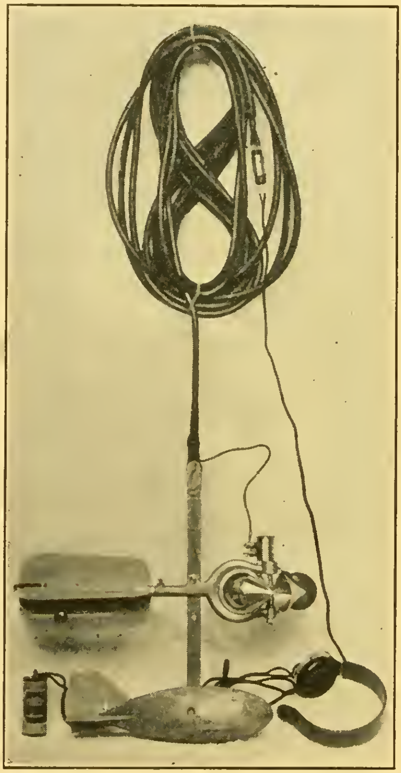


Fig. 72.—Price current meter. Latest model, showing improved yoke and new type of weight. (*Courtesy, W. & L. E. Gurley Co., Troy, N. Y.*)

The meter is either attached to a rod or suspended from a cable when being used, the latter being the usual method. When suspended by a cable, the cable is attached to a thin steel bar which passes through the frame of the meter and is attached to the frame by means of a pivot about which the meter is free to move in a vertical plane through an extreme angle of possibly 20 deg. either side of horizontal.

a thin rim whose width is equal to that of the vanes. This rim serves a three-fold purpose: (*a*) strengthens the vanes, (*b*) protects the vanes from grass and other floating material, and (*c*) acts as a shield against oblique velocities. The wheel is 5 in. in diameter and has 6 vanes, the pitch of the wheel being 2.3 ft. The axis revolves in the frame in bearings which are made of non-corrosive iridium and are almost frictionless. The rear end of the axis is provided with a gear which meshes with another gear attached to a vertical spindle running through the frame to a counterbox where there are counting gears. It is also equipped to permit the revolutions to be electrically signalled.

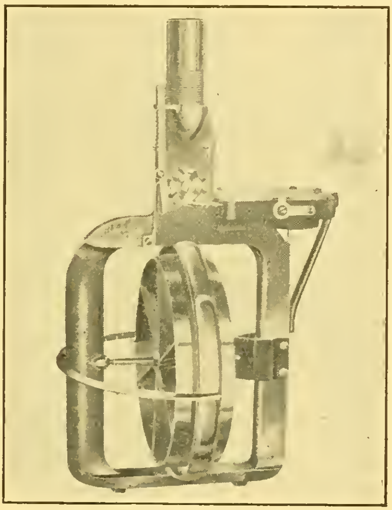


Fig. 75.—Fteley current meter.

The meter is always used at the end of a rod which is lowered into position and is supported in the hands of the observer. It is not suitable for measurements where the stream is deep and there is a high velocity because it is very difficult to hold securely in place.

117. Amsler Meter.—This meter (Fig. 76) consists of a propeller or two-bladed screw, which is made of cast aluminum and rotates about a horizontal axis. This axis is made of steel and is supported close to the head by a ball bearing. At the other end, the axis bears against a small steel pivot which takes up the thrust of the propeller. This pivot in turn bears on a sap-

phire. On the axis, there is a screw which engages with a system of gearing and rotates a cam once in fifty revolutions of the propeller. The cam makes a contact with a spring and closes the electrical circuit, thereby giving a bell signal, the current being supplied, normally, by two dry cells.

The frame of the meter, which is also made of cast aluminum, has a hole and clamping screws which permit the meter to be moved up and down on a rod and clamped at whatever depth is desired. The rod is generally ¾ or 1 in. in diameter and its point is driven into the bed of the stream.

There are two models of this meter. The larger of the two is provided with a tail and is known as the 1915 model. The later

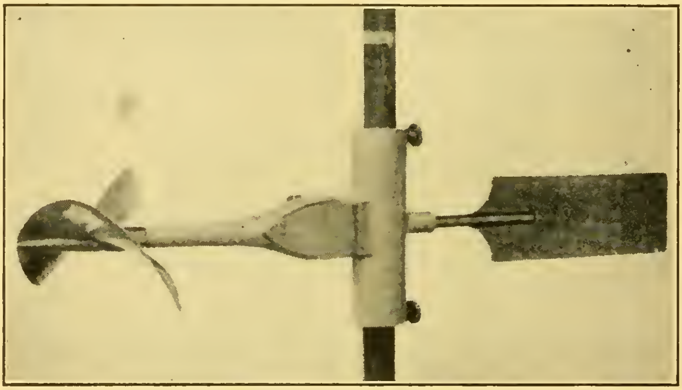


Fig. 76.—Amsler current meter. (*From Hogan's "Current Meters."*)

model has a smaller propeller than the 1915 model and is more compact, the tail being omitted. These meters have been used in European countries for some time and at present are in use to a small extent in the work being done by the French and the Swiss governments.

118. Ott Meter.—This meter (Fig. 77) is of European make, being manufactured by Ott and Kempten in Bavaria.[1] The meter is of the screw type. There are various ways of mounting the meter. It may be mounted on a standing rod placed on the bottom of the river, the meter being free to slide along the rod into position, or it may be suspended by a rod or cable. The cable suspension is suitable for use in deep and swift streams. A typical method of arrangement is to have the meter suspended

[1] Sold in U. S. A. by Keuffel and Esser Company, New York.

by a 2-in. tubing. An ordinary screw current meter is placed at its forward end and at its rear there is fastened a bulb-shaped enlargement and a four-bladed rudder. The instrument is suspended from a universal joint placed one-quarter of the distance from the forward end of the meter. This type of meter is always equipped with a ground contact plate for the purpose of giving notice when the meter touches the bottom.

A special feature for some of these meters is the so-called "Back Flow Indicator," which consists of a small flow pendulum fitted into an opening in the tail of the meter. Under normal flow conditions, the pendulum is deflected backwards, but when the flow is reversed, the pendulum is pushed forward and makes

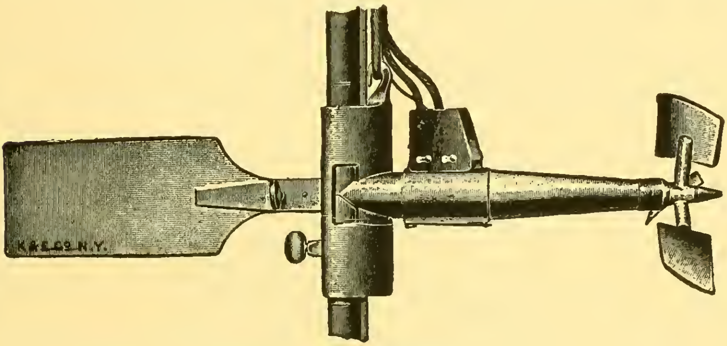


FIG. 77.—Ott current meter. (*Courtesy, Keuffel and Esser Co., New York.*)

an electrical contact and gives a signal that a reversal of flow has taken place. This finds its use in gaging in turbine head races where the flow in some parts of the cross-section is in the opposite direction to the main stream.

This meter has been used extensively by the governments of Germany, Russia, France, Spain, Argentina, and Chile.

119. Stoppani Meter.—This meter (Fig. 78) is manufactured by Stoppani, at Berne, Switzerland. It is essentially the same as the Ott meter having an additional guard ring around the blades.

The meter consists of a frame placed vertically and carrying the horizontal axis on two steel pivot bearings. On one end of the axis, there is a three-bladed propeller, and on the other, a worm which meshes with a worm gear and commutator. The commutator is provided with four bars and it rotates once in each hundred revolutions of the meter, indicating every twenty-

five revolutions. The contact system is freely exposed to the water, being protected by a loose guard. The propeller is surrounded by a slightly conical-shaped ring which protects the propeller from floating bodies and shields the meter from the effect of oblique velocities.

The meter is used with a rod which slides in a frame fastened to a rigid platform above the surface of the water. The meter

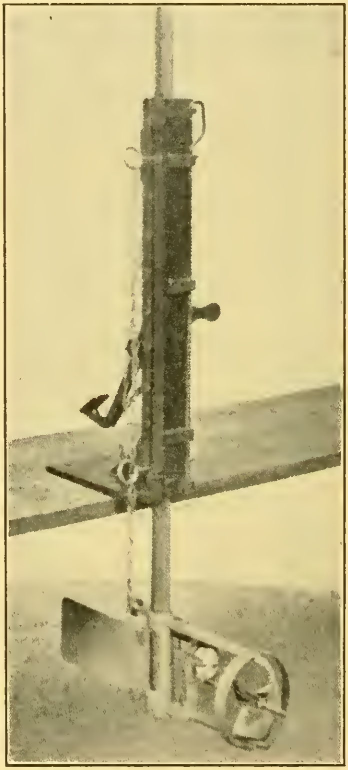


FIG. 78.—Stoppani current meter. (*From Hogan's "Current Meters."*)

is brought to any required position by releasing the spring clamp controlled by a pedal on the support.

120. Haskell Meter.—This meter[1] (Fig. 79) is similar in appearance to the Amsler meter previously described. Its screw terminates in a conical-shaped head designed to throw off any debris that might come against it. It is designed so that the action of the current pressure on it is integral and not differential.

[1] Manufactured by E. S. Ritchie and Sons, Brookline, Mass.

The meter is supported by a cable and swings in gimbals. It is used with a torpedo-shaped weight. Its tail consists of two blades at right angles to each other.

This meter has been used in studies on the Mississippi, the St. Lawrence, the St. Clair, and other large rivers in America, and by the Indus River Commission in India.

Fig. 79.—Haskell current meter.

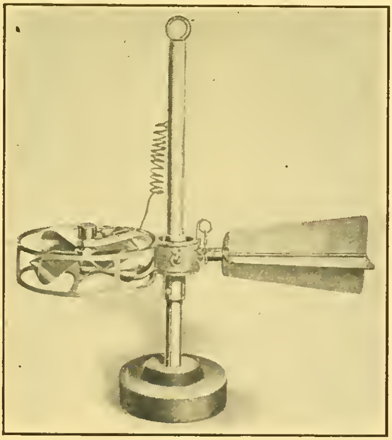


Fig. 80.—Ellis current meter.

121. Ellis Meter.—The Ellis meter[1] was devised by Gen. T. G. Ellis and was used by him in his measurements on the Connecticut River.

It is of the cup-shaped type, having four specially shaped buckets which are fastened to a vertical shaft. In the present model (Fig. 80), the wheel is surrounded by a cage for the purpose of

[1] Manufactured and sold by Buff and Buff, Boston.

protecting the buckets against being damaged by floating debris.

The upper end of the shaft terminates in a contact chamber fitted with electrical connections for use in observing the number of revolutions of the wheel.

At the opposite end of the frame, is the tail, composed of four blades fitted into the frame at right angles to each other.

The meter is supported by a rod and swings in gimbals. A weight is fastened to the lower end of the rod.

The present type of Ellis meter is principally intended for deeper river and sea work.

122. Methods of Supporting Meter.—The meter is supported by means of either a rod or a cable.

Rod Support.—There are two types of rod supports, the standing rod and the suspended rod. Of the standing rod, there are two forms, (*a*) with the meter fixed to the rod, and (*b*) with the meter free to slide along the rod. The former has the disadvantage of requiring the meter to be raised above the surface in order to alter its position on the rod, whereas with the latter, the meter slides along the main rod and is held by a small auxiliary rod which extends above the water surface and parallel to the main rod. The meter can be raised or lowered by moving the small rod.

With the suspended-rod type, the meter is fixed at the end of the rod and is held either by hand or by a bracket on a platform over the water surface.

Cable Support.—The cable requirements are (*a*) that it be strong enough to support the weight of the meter and sinker, yet not too large in cross-section to offer much resistance to the flow of water, and (*b*) that it be protected from possibility of short-circuits. The suspended meters are generally provided with a tail to keep the meter headed in the direction of the current and a torpedo-shaped lead to cause the meter to sink. The meter and tail are fastened together and the weight is placed below the meter and parallel to it. A thin rod connects the two and to this rod is fastened the cable.

There is a tendency for the meter to be carried by the current out of its apparent position so that for deep water and high velocities there must be provided a guy line to hold the meter in position. Increasing the weight of the lead will sometimes be sufficient to keep the meter hanging vertically.

CHAPTER IX

RATING THE CURRENT METER

123. Object of Rating.—When a current meter is used to determine the velocity of flow, two direct observations are made, namely,

1. The number of revolutions of the wheel as indicated electrically, acoustically, or by dials.
2. The corresponding time taken to register the revolutions.

From the data obtained by these observations, the rate of rotation of the wheel can be calculated and the corresponding velocity of the water can be deduced.

The object of rating a meter, therefore, is to make it possible to deduce the velocity of flow from the known speed of rotation of the wheel. This deduction is made by means of an experimental rating curve constructed from data obtained by several experimental ratings.

124. Methods of Rating.—A current meter is rated by comparing the rate of rotation of the wheel with a known velocity of flow past the meter. There are three general methods used in rating, namely, (*a*) the meter is towed through still water at a known speed; (*b*) the meter is used to measure the discharge past a section simultaneously with the measurement of the same discharge by more absolute methods; and (*c*) the velocity is measured alternately by means of a float and a meter.

The first method is the standard method of rating. It consists in towing the meter through a body of still water for a given distance at different known velocities and obtaining the corresponding speeds of rotation of the wheel. The velocities at which the meter is towed through the water are chosen to cover the range of velocities of the water which the meter is intended to measure. This method is considered in detail in the following articles.

The second method gives only an average relation between the speed of the wheel and the velocity of the water since the average velocity in the channel is obtained instead of the velocity at a

single point. The meter, in this method, is moved about through the cross-section, integrating the velocities at many points, and the average rate of rotation of the wheel is compared with the average velocity of flow.

The third method assumes an average velocity along the float course equal to the average velocity past the meter. There is no surety that such a relation holds and, at best, such a rating is only approximate.

125. Length of Course for Rating Meter.—In the standard method of rating, where the meter is towed through a still body of water at various known velocities, it is necessary that a channel of adequate length and cross-section be provided.

The distance covered by the runs are generally from 50 to 300 ft. A run shorter than 50 ft. will not give very accurate results. In addition to the measured course over which the timing is done, there should be a short run at the beginning in order that the wheel may be revolving at its proper speed at the beginning of the measurement. If it is permissible, a short run should also be provided at the end of the course to allow the meter to be brought to a stop gradually.

126. Size of Channel.—The cross-section of the channel should be large enough to permit the meter's being towed through the water without interference from any waves created by the motion of the meter.

A test made in the Berne channel which is 1.2 m. wide and 1.2 m. deep showed that at a certain velocity, usually around 2.5 m. per second, the number of revolutions made by the meter per metre of travel would start to diminish, reaching a minimum, and then increasing to normal at 3.5 m. per second. The cause of this variation in the relation of the revolutions of the wheel to the speed of motion through the water was attributed to the disturbance caused by the wave formed by the meter as it moved through the water.

This same phenomenon was observed at other stations where the channel was small. When the tank was large the phenomenon was not observed. This seemed to indicate that there is a critical range of velocities where the revolutions of the meter register lower than their true value. The velocity at which this lowering of the meter revolutions occurs is a function of the depth of the water in the channel and the amount of the error will depend upon the amount of the disturbance.

Tests to determine the effect of small channels on ratings have been carried on at the National Physical Laboratory at Torquay, England, and at the Imperial College, South Kensington, England. The channel of the Froude tank at the National Physical Laboratory was 30 ft. wide and 12¼ ft. deep, while the channel at the Imperial College station was 5 ft. wide and 3¼ ft. deep. Comparisons of the normal ratings made at the two places showed no serious difference between the results obtained in large and small channels. There was a general tendency for the velocity given by the Froude tank rating to be slightly greater than that given by the college rating for the same number of revolutions per second. In the case of a Price meter, so rated, this difference amounted to 0.45 per cent. It may be concluded that as a general rule the channel should be at least 4 ft. wide. Its depth should be at least 5 ft. to allow the meter to be held 2 ft. below the water surface and allow a 3-ft. clearance between the meter and the bottom of the channel.

127. Methods of Towing Meter.—The meter may be moved through the water in the following ways:

1. By suspension from a truck or car moving on a track placed along side of the water. The car may straddle the water and the meter hang from its center, or the car may run along one side and the meter hang from an arm extending over the water.

2. By suspension from the bow of a boat. When a boat is used it is propelled by a motor, tow-line, or oars. The oars are the least desirable because of the difficulty in keeping the velocity uniform. It is well to have the meter at least 5 ft. beyond the bow of the boat in order to be unaffected by any disturbance caused by the boat.

3. By suspension from the end of a horizontal arm, which describes a circle about a pivot, the meter thus passing over a course of known length.

128. Method of Supporting Meter.—When being rated, it is desirable that the meter be supported in the same way that it is to be supported in measuring. The rating curve for the meter when supported by a cable may not be applicable to measurements made with the rod support, and vice versa. Consequently, when a meter is sent to be rated the method of support should be stated.

In 1900 some experiments were made by E. G. Paul[1] to determine the difference in ratings of a Price meter when supported

[1] *U. S. Water Supply Paper* 95.

by a cable and by a rod. In these tests, it was found that the meter when supported by a rod turned at a faster rate for a given velocity than when supported by a cable. In other words, any given rate of rotation of the wheel will indicate a higher velocity when cable supported than when rod supported. Figure 81 shows a curve of percentage difference in the two velocities for any given rate of rotation of the wheel. It will be observed that the percentage difference decreases as the velocity increases.

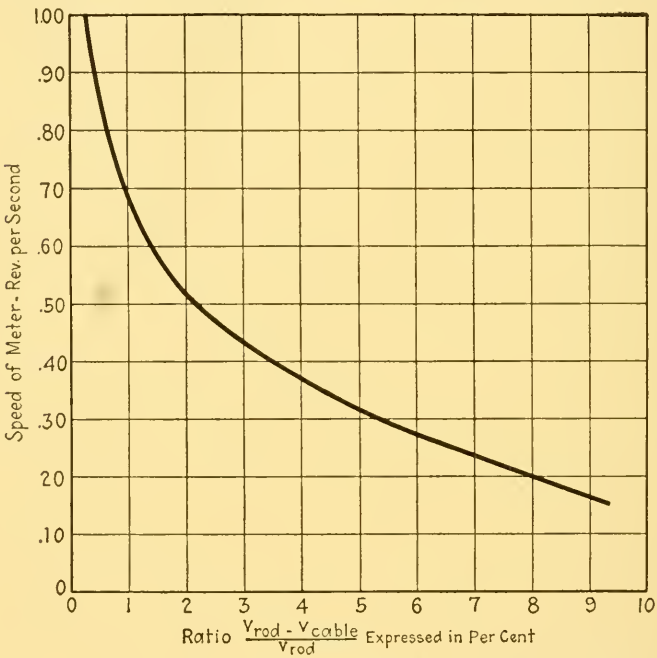


FIG. 81.—Curve showing percentage difference in velocity as shown by small Price meter No. 351 when rated in still water on a rigid rod and on a cable. (Constructed from table in U. S. Water Supply Paper 95, p. 89.)

The results of these tests should be considered of qualitative rather than quantitative value because in addition to the fact that the meter was cable supported, the size and position of the weight would have an effect on the rotation of the meter. Hence we may safely say that there may be a difference in the ratings of a meter when cable supported and when rod supported and that the percentage difference will increase as the velocity measured decreases.

The fact that the size and position of the weight will affect the rating of a meter has recently been shown by tests made at

the U. S. Bureau of Standards. Figure 82 shows the results of these tests. The standard suspension is No. 2 with the meter at the center and a 15-lb. weight below the meter. For this suspension, the coefficient is unity. The coefficients for other suspensions are shown in the other sketches.

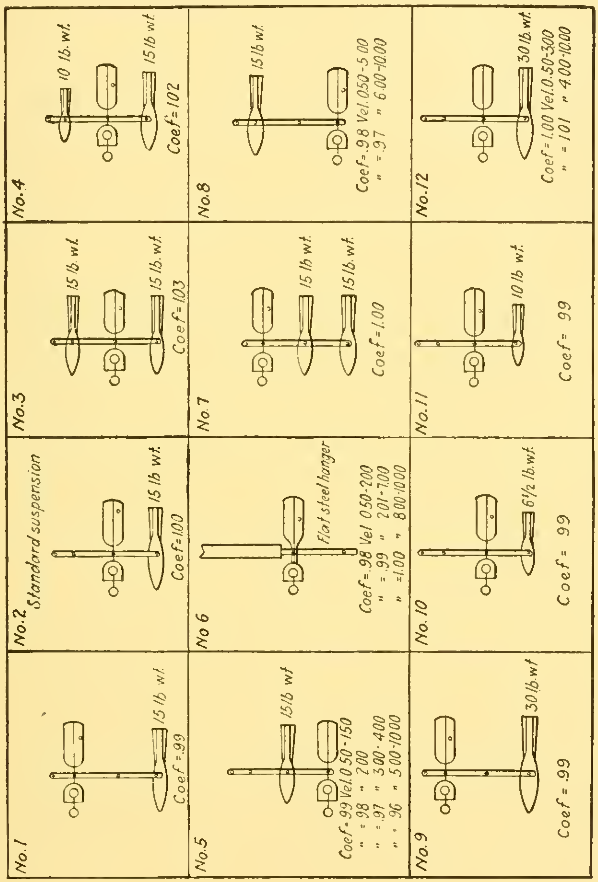


Fig. 82.—Suspension coefficients for current meters. Showing effect of relative positions of the meter and the weight. The coefficient for each arrangement of the meter and weight, or weights, is based on suspension No. 2 whose coefficient is taken as 1.00. These twelve tests were, made at the U. S. Bureau of Standards for the Water Resources Branch of the U. S. Geological Survey and are reproduced here by permission of the District Engineer's Office at Boston, Mass.

129. Methods of Moving Car.—The car which carries the meter may be propelled either by hand or by a motor. The car when propelled by a motor has undoubtedly a more reliable uniform rate of travel than when pushed by hand but even this cruder method may be perfected after a little practice so as to give sufficiently accurate results.

The rate of travel can be kept reasonably uniform by the aid of marks placed every 5 or 10 ft. along the track and having the seconds called off as the car is moved along the track. In this way the car can be made to pass the marks at regular intervals of time.

This method of hand propulsion is used at the Imperial College at South Kensington, England, and the Canadian Station in Alberta.

The motors used to propel the car are of different types. At the Swedish Station at Horsborg a constant speed motor with a centrifugal governor is employed. At the Froude tank, two types are used. One is a steam engine with a sensitive centrifugal governor. The other consists of four direct current motors driven by a motor generator and worked either in series or in parallel. The generator gets its current from one set of storage batteries and a second set supplies the field current. The speed of the car can be varied over a wide range by varying the field of the motor generator set. At all speeds the car runs with a uniform motion. At the Bureau of Standards at Washington, the motor drives the car through a Waterbury hydraulic transmission which regulates the speed.

130. Rating Stations.—There are various types of rating stations. Some are designed especially for rating meters while others were originally intended to be used for other purposes but have been rigged up later to serve as rating stations.

The necessary features of a rating station are (*a*) suitable body of water, (*b*) means for controlling course of meter through water, (*c*) car for towing the meter, (*d*) means for propulsion of car at uniform rate, (*e*) means for timing.

In most stations, the car runs on two rails, one on either side of the channel. In a few stations, either one or two rails are placed on one side of the channel and an arm which carries the meter extends from the car over the water. The Canadian Station is an example of this arrangement. At the Swedish Station the car is suspended from an overhead rail which was installed primarily for use in carrying sand in and out of the filter.

The record of the time, distance, and number of revolutions is best obtained by means of a chronograph arranged so as to be operated by, and move with, the main driving wheels. The time, distance, and number of revolutions can be recorded on the revolving drum.

Among the several types of stations which are used, the following may be cited:

Naval Tanks.—Froude tank of the National Physical Laboratory at Torquay, Devon, England; University of Michigan, Ann Arbor, Michigan; and Berlin, Germany.

Existing Pool of Water.—Egyptian Public Works at Abbassiya, a reservoir; Swedish Waterfalls Administration at Horsborg, a filter bed.

Revolving Arm.—French Service des Force Hydrauliques at Grenoble; Worcester Polytechnic Institute at Worcester, Massachusetts.

Specially Constructed Stations.—United States Bureau of Standards, Washington, D. C.; Rennselaer Polytechnic Institute, Troy, New York; Cornell University, Ithaca, New York; Canadian Irrigation Office, Alberta; Imperial College, South Kensington, England; and the Swiss Hydrometric Survey at Berne, Switzerland.

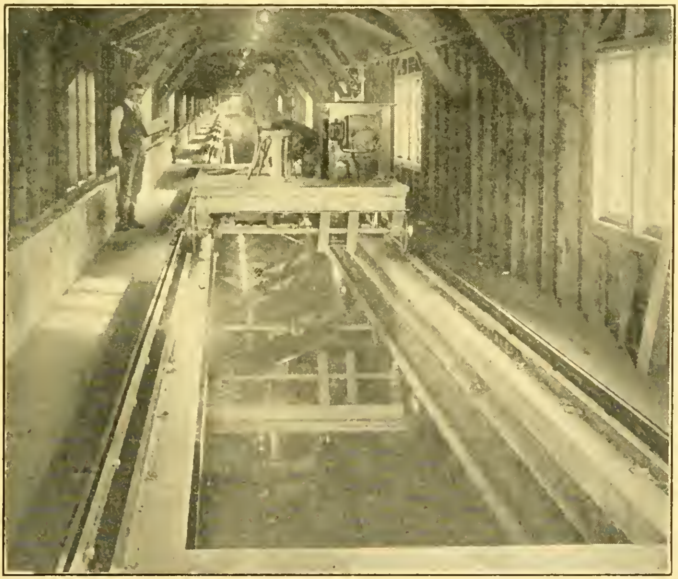


Fig. 83.—View of current meter rating station, U. S. Bureau of Standards, Washington, D. C. (*Courtesy, U. S. Bureau of Standards.*)

131. Rating Station at the United States Bureau of Standards. The United States Bureau of Standards rating station at Washington consists of a channel 400 ft. long, 6 ft. wide, and 6

ft. deep. The channel is made of reinforced concrete. Extending the entire length on either side of the channel are two rails, one on which the wheels of the car run and the other a "live" rail used in supplying the driving motor with current. On either side of the channel and extending the entire length, there is a walk for the convenience of the recorder. Everything is housed in a wooden shelter (Fig. 83).

The rating car is driven by a motor which runs at a constant speed. The speed is controlled entirely by means of a Waterbury hydraulic gear. The variation in speed is not accomplished by steps but the speed range is made continuous from the minimum to the maximum. The car is brought approximately to any desired speed by means of the controller handle of the speed gear which moves over a scale graduated in feet per second. The actual velocity of the car, however, is obtained from measurements of the time and distance traveled by the car.

The method of rating a meter consists in making nine or ten double runs at various velocities ranging from about $\frac{1}{2}$ to $7\frac{1}{2}$ ft. per second. These are the general limits of the range covered but velocities as low as $\frac{1}{10}$ ft. per second and as high as 20 ft. per second can be measured with the apparatus.

The average of each pair of runs is used in the determination of points to be used in constructing the rating curve. After the curve has been drawn through the plotted points, the rating equation or equations are deduced and the rating table prepared.

Reports on the rating of a meter, sent to the Bureau to be rated consist of a statement of the rating equations, together with the conditions of rating and a blueprint of the rating curve. Rating tables are not furnished by the Bureau.

132. Description of Rating Apparatus Used at the U. S. Bureau of Standards.—The following description of the rating apparatus used at the U. S. Bureau of Standards is taken from information furnished by the Bureau.

CURRENT METER RATING STATION

Description of the Apparatus Used in the Rating of Water Current Meters.

The rating apparatus consists essentially of three parts, (*a*) a revolution counter operated by the electric contacting device in the meter itself, (*b*) a device for discharging arrows into a graduated board extending along the flume, operated by the revolution counter, and (*c*) a stop-watch operated by the arrow discharging device.

A diagram of the apparatus showing the electrical connections is shown in Fig. 84. Beginning at the left of the drawing, with the meter, it will be seen that a relay, R, is introduced in the signal circuit which will close a circuit through the magnet E_2 at each closure of the signal circuit in the meter itself. To the armature of the magnet, E_2, is attached a pawl which engages with teeth cut around the circumference of the wheel, W, which serves as a revolution counter. Each signal from the meter advances the wheel, W, one tooth. There is a total of 40 teeth on the wheel so that a complete revolution of the wheel would correspond to 40 revolutions of a meter having a commutator making contact with each revolution. The relay, R, is introduced to permit the use of only a very small current through the meter itself. The switch, N, serves to start or stop the revolution counter at will, independently of the meter.

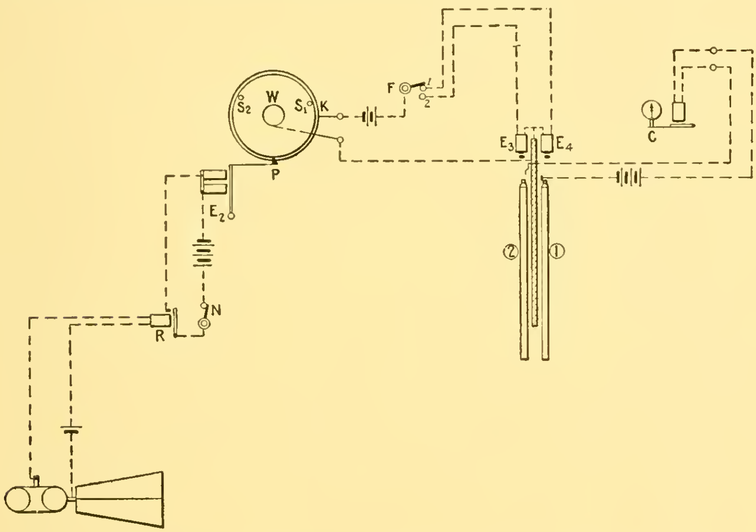


Fig. 84.—Diagram of electrical connections for meter rating car. U. S. Bureau of Standards. (*Courtesy, U. S. Bureau of Standards.*)

The device for shooting the arrows, for the measurement of distance traversed, is shown in Fig. 85. The discharge of the arrows is effected automatically, one at the beginning and one at the end of each run by the revolution counter. The pins, S_1, and, S_2, (Fig. 84) are set in the brass revolution counterwheel, W, to include a given number of the teeth cut on the circumference, or, in other words, for a run of a given number of revolutions of the counter. As the wheel revolves (clockwise as shown) one tooth at a time, the pin, S_1, approaches and then comes into contact with the brush, K, closing the circuit through the operating switch, F-1, and the magnet, E_4, which then discharges arrow 1. When the wheel, W, has advanced so that contact between the pin, S_1, and the brush, K, has been broken, the switch, F, is moved to point 2. The wheel, W, continuing to

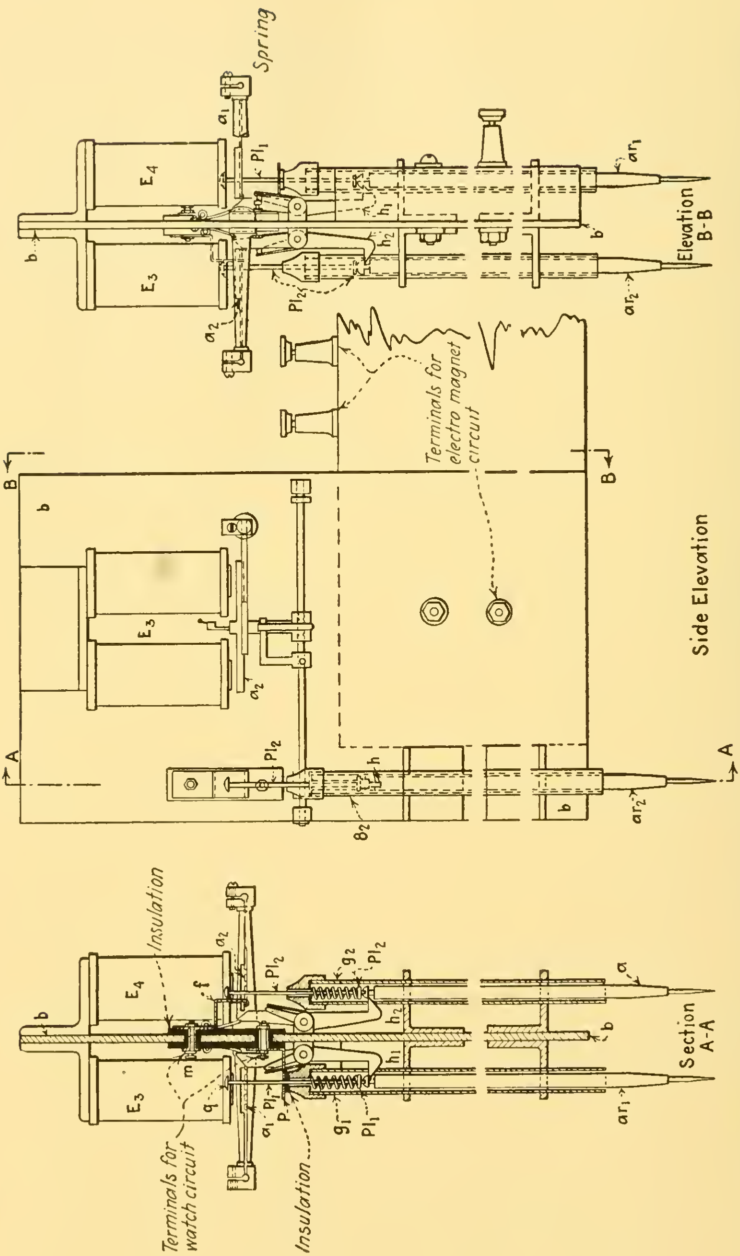


FIG. 85.—Diagram of device for discharging arrows from meter rating car. U. S. Bureau of Standards. (*Courtesy, U. S. Bureau of Standards.*)

advance one tooth for each contact in the meter, at the end of the predetermined number of revolutions of the meter, brings the second pin S_2 into contact with the brush, K, closing the circuit through switch, F-2, and magnet, E_3, releasing arrow 2.

The stop-watch, C, is automatically operated by the arrow discharging device. Referring to Fig. 85, the arrows, ar_1, and, ar_2, are held in place in the guns, against the spring-actuated plungers, Pl_1, and, Pl_2, with projecting stems, q, by the catches, h_1 and, h_2, engaging the underside of the steel screw heads, attached to the brass shafts of the steel pointed arrows. When the first arrow is discharged, as described above, the circuit through the magnet operating the stop-watch, C, is closed, starting the watch. This circuit remains closed, the watch running until released by the opening of the circuit by the discharge of the second arrow. The details of the arrangement by which this is effected are shown in the section at the left (Fig. 85). When both arrows are in place in the guns, the plungers above the arrows are both in the up position projecting from the guns. The watch circuit through the arrow discharging device is so arranged that the plunger over the arrow ar_1, the first to be discharged (at the left in the section drawing), closes the circuit when in the down position, the plunger over arrow, ar_2, when in the up position. Both plungers being up the circuit is open. When the first arrow is discharged by the gun, g_1, and the plunger, Pl_1, descends, forcing the nut, q, against the metal bracket, p, the circuit is completed as follows: From terminal, m, through the screw piercing the insulation in the supporting frame, to contact spring, f, through the guns, to plunger, Pl_1, through the brass bracket, p, to the second terminal. When the second arrow, ar_2, is discharged at the end of the run and the plunger, Pl_2, above it descends the circuit is opened by the head of the plunger losing contact with the spring, f.

The operation of taking a run is as follows : The operator is seated on the car facing the instrument board on the table. The meter suspended through the opening shown in the car platform is in full view. The car control is within reach of his right hand. Both arrows are inserted in the guns and the pins, S_1, and, S_2, are set in the wheel of the revolution counter, W, for a number of revolutions corresponding to the length of run desired. The operating switch, F, is placed on the first point, the revolution counter switch, N, is open. The operator then moves the car controller handle slowly to the point on the graduated scale on the controller corresponding approximately to the speed desired. When the desired uniform speed has been attained, the switch N is closed and the revolution counter advances the pin, S_1, until it comes in contact with K. This contact shoots the first arrow and starts the watch. After S_1 has broken contact with the brush, K, the switch, F, is moved to point 2. At the end of the run the second pin, S_2, makes contact with K, shooting the second arrow and stopping the watch. The operator then brings the car to a stop by moving the controller to the neutral position. An assistant walking along the flume notes the numbers on the graduated board where the arrows have been discharged, the difference of which give the distance traversed for the number of revolutions for which the revolution counter was set, in the time indicated by the stop watch on the instrument board in front of the operator. A run in the opposite direction is made as described above by moving the car controller in the reverse direction.

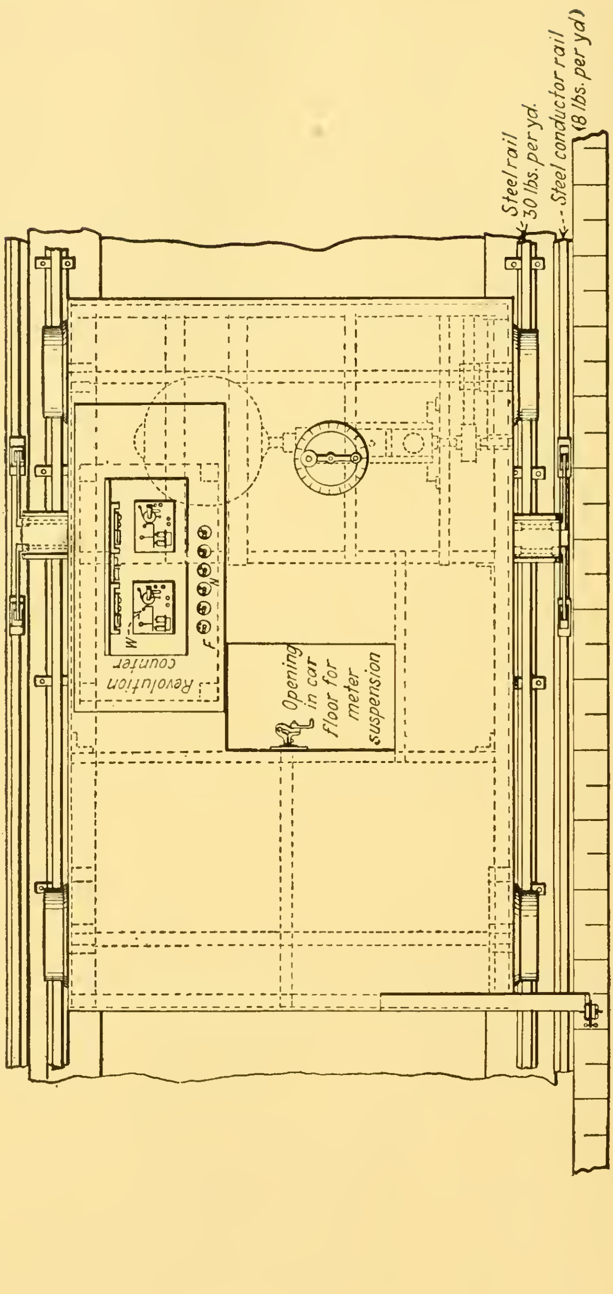


FIG. 86A.—Plan view, meter rating car. U. S. Bureau of Standards. (*Courtesy, U. S. Bureau of Standards.*)

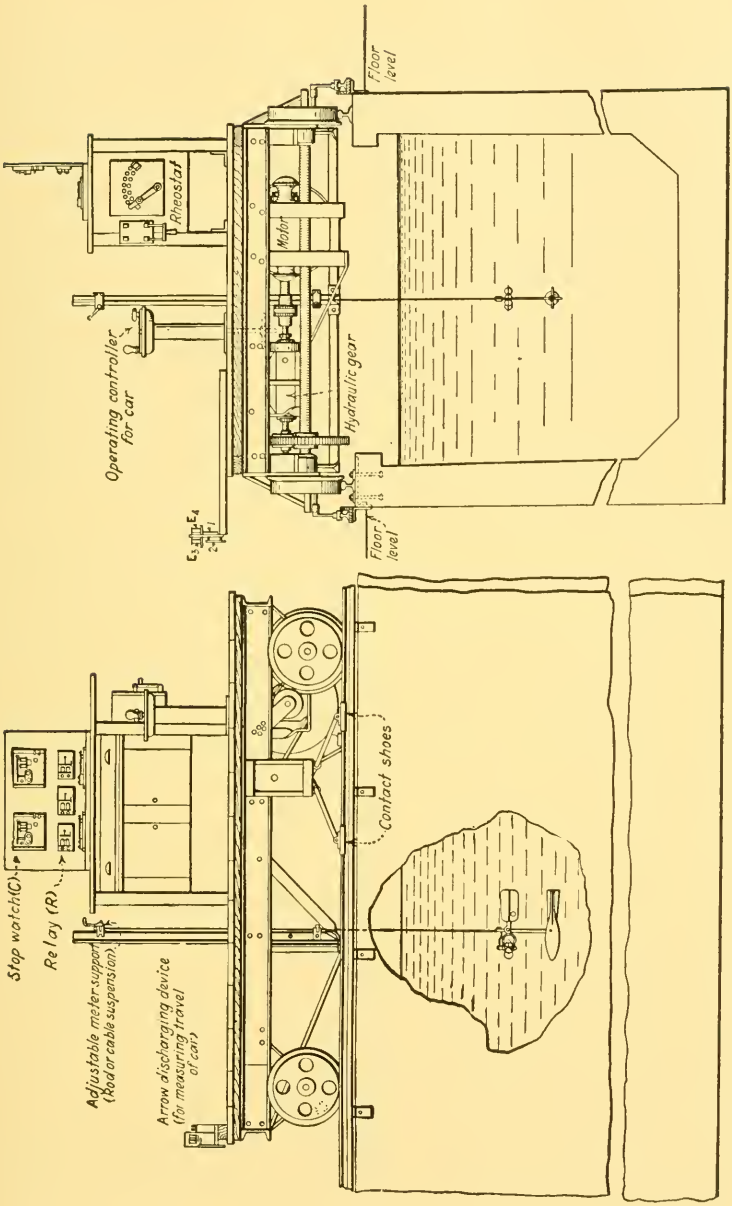


FIG. 86B.—Side and end elevations, meter rating car. U. S. Bureau of Standards. (Courtesy, U. S. Bureau of Standards.)

133. Current Meter Rating Curve.—In passing over a given distance the meter wheel should revolve, theoretically, the same number of times regardless of the velocity of motion through the water. The theoretical curve showing the relation between the speed of rotation of the wheel and the velocity of motion would then be a straight line. However, due to such factors as inertia of the moving wheel, friction of the bearings, slip of the blades, and the retarding effect of the pressure on the convex side of the cups, the relation between the speed of the wheel and the velocity of motion is not constant for all velocities. As a consequence, the rating curve is not actually a straight line. However, the effect of the friction, inertia, etc. decreases proportionately with an increase in velocity so that only at very low velocities, say, below 0.5 ft. per second, is the curved markedly different from a straight line. Above 0.5 ft. per second the curve approaches a straight line and for all practical purposes may be considered as straight.

The rating curve will not pass through the origin of coordinates because the wheel does not commence to rotate until the meter has acquired a velocity of 0.1 to 0.5 ft. per second, due to the resistance to rotation offered by the factors mentioned above. The point of intersection of the curve and the axis of velocities will be at that velocity at which the wheel starts to revolve.

134. Construction of Rating Curve.—The rating curve is constructed by plotting the observations on coordinate paper, generally letting the revolutions per second be the ordinates and the velocities be the abscissas. As has been said, for velocities above 0.5 ft. per second, the points will fall on or very close to a straight line. Points which fall much away from such a line should be disregarded in drawing the curve.

The mean of all the observations should be plotted and a straight line drawn through this point and in such a direction as to average all the other plotted points. In some cases, two straight lines of different slopes will better fit the plotted points and should be drawn instead of the single line.

Figure 87 shows a rating curve obtained from a set of ratings of a small Price meter made at the U. S. Bureau of Standards. Nine pairs of runs were made, the velocities ranging from about 0.5 ft. per second to about 3.5 ft. per second. For velocities below 1.0 ft. per second the curve is drawn to a larger scale. This permits of closer reading of the curve for the low velocities.

Below 0.5 ft. per second no information was obtained. Very probably the line would intersect the velocity axis around 0.2 ft. per second.

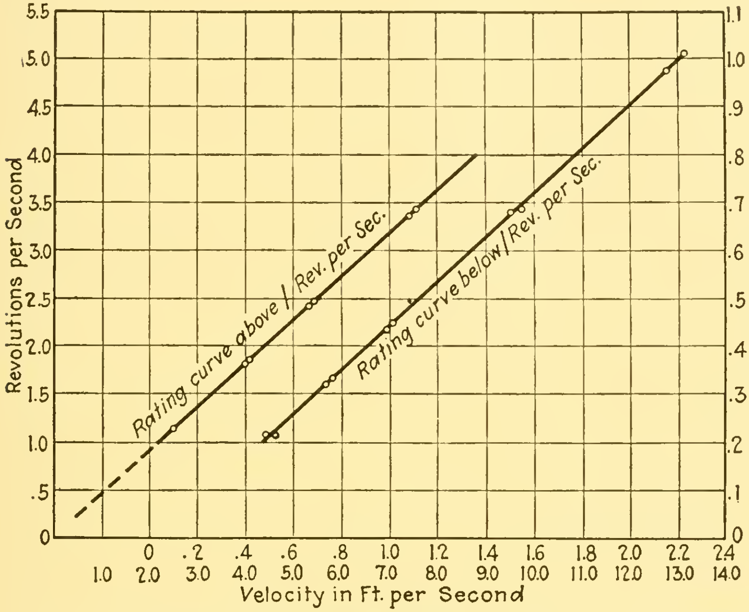


FIG. 87.—Rating curves for Price meter P214010. Rated at the U. S. Bureau of Standards, Washington, D. C.

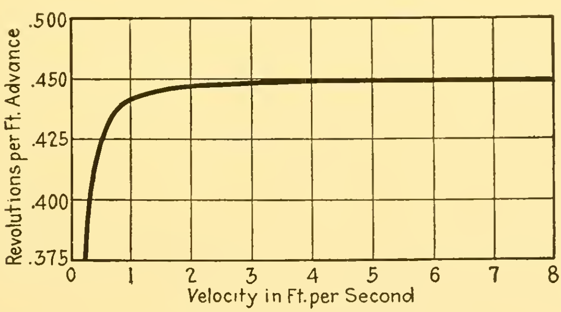


FIG. 88.—Rating curve for a Price current meter.

Another form of rating curve is sometimes used which is, in some respects, preferable. It is the method of plotting the number of revolutions per foot of advance against the velocity (Fig. 88). Such a method emphasizes any experimental errors.

As each observation is made, it can be plotted to see if it is satisfactory or should be repeated. Theoretically, the line should be straight and parallel to the axis of coordinates of velocity. Again, due to slip and friction, it will not become parallel to the velocity axis until a velocity of 0.5 ft. or more per second is reached. Below 0.5 ft. per second the line will curve down toward the velocity axis. The relation between the speed, n, and the velocity, v, can be obtained by first multiplying the number of revolutions per foot by the velocity to obtain n.

135. Equation of Rating Curve.—The equation of the rating curve may be written in any one of three forms depending on the assumption made as to the shape of the curve. The three forms are as follows:

$$(1) \qquad v = an + b,$$
$$(2) \qquad v = an^2 + bn + c,$$
$$(3) \qquad v^2 = an^2 + b,$$

where v and n are respectively the velocity and the number of revolutions per second, and a, b, and c are constants. The value of the constants, b, in (1) and (3) and c, in (2) will depend on the velocity at which the wheel starts to revolve.

Equation (1), being that of a straight line, does not fit the conditions as accurately as equations (2) and (3), but, because of its simplicity and the negligible error introduced by its use, it is commonly taken as the equation of a rating curve.

In deriving a curve for the Price meter, for ranges in velocities from 0.3 to 10 or 12 ft. per second Parker[1] suggests that the curve be considered in two parts, taking

$$v = a_1n + b_1 \text{ up to } v = 3.5 \text{ to } 4 \text{ ft. per second}$$

and

$$v = a_2n + b_2 \text{ for greater velocities.}$$

In the case of the Fteley meter, Diamant[2] has suggested that for values greater than $v = 1.2$ ft. per second, the form $v = an + b$ holds fairly well, but for smaller velocities, the form $v = \frac{a_1}{n} + b$ is more accurate.

The idea of using two equations to express the relation of the velocity and speed of the wheel is found by the U. S. Bureau of Standards to be practicable and as a general rule two equations

[1] Parker, "Control of Water," Geo. Routledge and Sons, Ltd.

[2] *Trans. Am. Soc. Civ. Eng.*, vol. 66.

are furnished with each rating made by the Bureau. For example, Fteley meter, 8932, rated for the Massachusetts Institute of Technology was found to require two equations to express the relations between the velocity and the speed of the wheel. They were as follows:

n less than 2.00,

$$v = 0.900n + 0.045;$$

n greater than 2.00,

$$v = 0.943n - 0.040.$$

Again, a rating of Fteley meter, 6692, showed

n less than 1.10,

$$v = 0.923n + 0.085;$$

n greater than 1.10,

$$v = 1.000n.$$

On the other hand, a rating of Price meter, 1986, showed,

$$v = 2.190n + 0.030$$

for all velocities.

The equation which expresses the most probable relation between the velocity and the revolutions per second of the wheel is obtained by the method of least squares. First, however, the form of the equation has to be assumed. The form may be any one of those given earlier in this article. The straight line form will be used to illustrate the method, as this form is the one most generally assumed in practice.

The equation is $\qquad v = an + b$

which may be written $\qquad v - an - b = 0.$

For actual observations, unless they are perfect, the expression, $v - an - b$ will have some value other than zero. This value is known as the residual error and the equation may be written

$$v - an - b = r$$

where r is the residual.

According to the principle of least squares, the most probable values of the constants a and b will be such as will make the sums of the squares of the residuals a minimum.

For m observations there will be m equations. Squaring each of these equations and adding,

$$\begin{aligned}\Sigma r^2 &= r_1^2 + r_2^2 + r_3^2 + \cdots r_m^2 \\ &= [v_1 - an_1 - b]^2 + [v_2 - an_2 - b]^2 + [v_3 - an_3 - b]^2 \\ &\qquad \cdots + [v_m - an_m - b]^2.\end{aligned}$$

Differentiating with respect to a and b and placing the resulting expressions equal to zero,

$$d\frac{\Sigma r^2}{da} = -2n_1[v_1 - an_1 - b] - 2n_2[v_2 - an_2 - b] - 2n_3[v_3 - an_3 - b]$$
$$\cdots -2n_m[v_m - an_m - b] = 0.$$
$$d\frac{\Sigma r^2}{db} = -2\,[v_1 - an_1 - b] - 2\,[v_2 - an_2 - b] - 2\,[v_3 - an_3 - b]$$
$$\cdots -2\,[v_m - an_m - b] = 0.$$

These two equations may be written in the form

$$a\Sigma n^2 + b\Sigma n = \Sigma\,(v \cdot n).$$
$$a\Sigma n + m \cdot b = \Sigma v.$$

Solving these equations simultaneously will give the values of a and b.

To illustrate this method, the most probable equation will be derived from a rating of a Fteley meter, belonging to the Massachusetts Institute of Technology. The work is best carried out in tabular form, as follows:

Number of observation	R. P. S. n	Velocity v	n^2	vxn
1	1.579	1.552	2.493	2.451
2	2.146	2.132	4.605	4.575
3	2.502	2.472	6.200	6.185
4	2.707	2.670	7.328	7.228
5	2.802	2.762	7.851	7.739
6	3.390	3.370	11.492	11.424
7	4.530	4.440	20.521	20.113
8	4.700	4.700	22.090	22.090
9	4.920	4.870	24.206	23.960
Totals.	29.216	28.968	106.786	105.765

Substituting the values of the several summations,

$$ax106.786 + bx29.216 = 105.765.$$
$$ax29.216 + bx9 = 28.968.$$

Solving simultaneously,

$$a = 0.982.$$
$$b = 0.032.$$

The equation is, therefore,

$$v = 0.982n + 0.032.$$

136. Constancy of Equations.—It is tacitly assumed, in the use of a rating curve, that the equation of the curve is constant. This assumption will be true if the condition of the meter remains the same as it was at the time of rating. But, if the bearings become worn or rusty, or if silt gets into the bearings, or the cups are seriously damaged or the pivot of the wheel is bent, the equation will be different and the meter will have to be rated again.

To illustrate the change which is liable to occur, the following ratings of a Price meter belonging to the Massachusetts Institute of Technology are given.

Table IX.—Results of Three Ratings of Price Meter 1067

Date	Number of observations	Velocity, feet per second	Meter coefficients	
			a	b
Mar. 3, 1914	10	0.78 to 5.63	2.23	0.04
June 28, 1918	13	1.73 to 6.00	2.32	0.13
July 5, 1923	14	0.70 to 5.72	2.15	0.06

This change in the values of the coefficients would make considerable error in the measurement of low velocities. Although the ratings were made 4 and 5 years apart, the meter was used in not more than twelve measurements a year. This would indicate that a meter should be rated at rather frequent intervals especially if the water is silty or the meter is subject to hard usage.

136a. Rating Table.—A rating table may be prepared from the observations in one of two ways. The first is the simpler of the two and consists in drawing the rating curve in the manner previously explained and then reading the velocities from the curve for each one-tenth of a revolution. The second method is to determine the most probable equation of the curve and then compute the velocities for each one-tenth of a revolution. For low velocities, this second method is necessary if an accuracy of 1 per cent is desired. Its disadvantage lies in the time that it requires.

The data may be entered in the table in one of two ways, *viz.*, (a) corresponding to revolutions per second, or (b) corresponding to total time and total revolutions. In the first method (Fig. 89), the revolutions per second of the wheel have to be computed

for each observation and the corresponding velocity obtained by interpolation in the table.

In the second method (Fig. 90), the time covered by the table is generally from 40 to 70 sec. and the revolutions from 5 to 100. The time is given for every second and the revolutions for every

Revs. per sec.	Vel. ft. per sec.	Revs. per sec.	Vel. ft. per sec.	Revs. per sec.	Vel. ft. per sec.	Revs. per sec.	Vel. ft. per sec.	Revs. per sec.	Vel. ft. per sec.
0.00	......	1.00	0.940	2.00	1.85	3.00	2.79	4.00	3.75
0.05	......	1.05	.982	2.05	1.90	3.05	2.83	4.05	3.79
0.10	0.170	1.10	1.03	2.10	1.95	3.10	2.88	4.10	3.84
0.15	.212	1.15	1.07	2.15	1.99	3.15	2.93	4.15	3.89
0.20	.255	1.20	1.12	2.20	2.04	3.20	2.98	4.20	3.94
0.25	.298	1.25	1.16	2.25	2.08	3.25	3.03	4.25	3.99
0.30	.341	1.30	1.21	2.30	2.13	3.30	3.07	4.30	4.03
0.35	.384	1.35	1.26	2.35	2.18	3.35	3.12	4.35	4.08
0.40	.426	1.40	1.30	2.40	2.22	3.40	3.17	4.40	4.13
0.45	.469	1.45	1.35	2.45	2.27	3.45	3.22	4.45	4.18
0.50	.512	1.50	1.39	2.50	2.31	3.50	3.27	4.50	4.23
0.55	.555	1.55	1.44	2.55	2.36	3.55	3.31	4.55	4.27
0.60	.598	1.60	1.49	2.60	2.41	3.60	3.36	4.60	4.32
0.65	.640	1.65	1.53	2.65	2.45	3.65	3.41	4.65	4.37
0.70	.683	1.70	1.58	2.70	2.50	3.70	3.46	4.70	4.42
0.75	.726	1.75	1.62	2.75	2.55	3.75	3.51	4.75	4.47
0.80	.769	1.80	1.67	2.80	2.60	3.80	3.55	4.80	4.51
0.85	.812	1.85	1.72	2.85	2.64	3.85	3.60	4.85	4.56
0.90	.854	1.90	1.76	2.90	2.69	3.90	3.65	4.90	4.61
0.95	.897	1.95	1.81	2.95	2.74	3.95	3.70	4.95	4.66

FIG. 89.—Rating table for Fteley current meter, No. 8932.

multiple of 10, with the exception of 5 which is useful for low velocities. The use of such a table requires the total number of seconds elapsed during the observation and the total number of revolutions counted in that time. With these two data, the corresponding velocity may be read directly from the table.

Secs.	Velocity in feet per second													Secs.
	5	10	20	30	40	50	60	70	80	90	100	150	200	
	Revolutions													
40	0.31	0.58	1.13	1.68	2.23	2.78	3.34	3.90	4.45	5.01	5.55	8.34	11.12	40
41	0.30	0.57	1.10	1.64	2.18	2.71	3.26	3.81	4.34	4.89	5.43	8.14	10.85	41
42	0.30	0.56	1.07	1.60	2.13	2.65	3.18	3.72	4.24	4.77	5.30	7.95	10.59	42
43	0.29	0.54	1.05	1.56	2.08	2.59	3.11	3.63	4.14	4.66	5.18	7.77	10.34	43
44	0.28	0.53	1.03	1.53	2.03	2.53	3.04	3.55	4.04	4.55	5.06	7.59	10.10	44
45	0.28	0.52	1.01	1.50	1.99	2.48	2.97	3.47	3.95	4.45	4.95	7.42	9.87	45
46	0.28	0.51	0.99	1.47	1.95	2.43	2.90	3.39	3.87	4.35	4.84	7.26	9.65	46
47	0.27	0.50	0.97	1.44	1.91	2.38	2.84	3.32	3.79	4.26	4.74	7.11	9.45	47
48	0.26	0.49	0.95	1.41	1.87	2.33	2.78	3.25	3.71	4.17	4.64	6.96	9.25	48
49	0.26	0.48	0.93	1.38	1.83	2.28	2.72	3.18	3.63	4.09	4.54	6.81	9.06	49
50	0.26	0.47	0.91	1.35	1.79	2.23	2.67	3.12	3.56	4.01	4.45	6.67	8.89	50
51	0.25	0.46	0.90	1.32	1.75	2.19	2.62	3.06	3.49	3.93	4.36	6.54	8.72	51
52	0.25	0.46	0.88	1.29	1.72	2.15	2.57	3.00	3.42	3.85	4.28	6.42	8.56	52
53	0.24	0.45	0.86	1.27	1.69	2.11	2.52	2.94	3.36	3.78	4.20	6.30	8.40	53
54	0.24	0.44	0.85	1.25	1.66	2.07	2.47	2.83	3.30	3.71	4.12	6.18	8.24	54
55	0.24	0.43	0.83	1.23	1.63	2.03	2.43	2.83	3.24	3.64	4.05	6.07	8.09	55
56	0.23	0.43	0.82	1.21	1.60	1.99	2.39	2.78	3.18	3.58	3.98	5.96	7.95	56
57	0.23	0.42	0.80	1.19	1.57	1.96	2.35	2.73	3.12	3.52	3.91	5.86	7.81	57
58	0.22	0.41	0.79	1.17	1.54	1.93	2.31	2.68	3.07	3.46	3.84	5.76	7.68	58
59	0.22	0.41	0.78	1.15	1.51	1.90	2.27	2.63	3.02	3.40	3.77	5.66	7.55	59
60	0.22	0.40	0.77	1.13	1.48	1.87	2.23	2.59	2.97	3.34	3.71	5.56	7.42	60
61	0.22	0.39	0.75	1.11	1.46	1.84	2.19	2.55	2.92	3.29	3.65	5.47	7.30	61
62	0.21	0.39	0.74	1.09	1.44	1.81	2.16	2.51	2.87	3.24	3.59	5.38	7.18	62
63	0.21	0.38	0.73	1.07	1.42	1.78	2.13	2.47	2.82	3.19	3.53	5.30	7.07	63
64	0.21	0.38	0.72	1.05	1.40	1.75	2.10	2.43	2.77	3.14	3.48	5.22	6.96	64
65	0.20	0.37	0.71	1.03	1.38	1.72	2.07	2.39	2.73	3.09	3.43	5.14	6.85	65
66	0.20	0.37	0.70	1.02	1.36	1.69	2.04	2.35	2.69	3.04	3.38	5.06	6.75	66
67	0.20	0.36	0.69	1.01	1.34	1.66	2.01	2.32	2.65	2.99	3.33	4.98	6.65	67
68	0.28	0.36	0.68	1.00	1.32	1.64	1.98	2.29	2.61	2.95	3.28	4.91	6.55	68
69	0.19	0.35	0.67	0.99	1.30	1.62	1.95	2.26	2.57	2.91	3.23	4.84	6.45	69
70	0.19	0.35	0.66	0.98	1.28	1.60	1.92	2.23	2.53	2.87	3.18	4.77	6.36	70

FIG. 90.—Rating table for Price current meter, No. 1986.

CHAPTER X

CURRENT METER CHARACTERISTICS

137. Conditions Affecting Use of Still-water Rating Curve.—In using a still-water rating curve to deduce the velocity of flow by means of an observed speed of rotation of a current meter, it is assumed that like conditions prevail in the stream as prevailed in the rating channel. In view of the earlier consideration of the conditions existing in flowing water, it is apparent that this assumption is not valid. The water in the rating channel is still and the rotation of the wheel is caused by a uniform movement of the meter through the water. This relative motion of the water and the wheel simulates steady and parallel or stream-line flow, whereas in the stream, the flow is not steady and is sinuous or turbulent.

The magnitude of the error resulting from the use of the still-water rating curve to obtain the velocity of flow will depend on the characteristics of the current meter and the degree of turbulency of the water. It is such characteristics as affect the accuracy of a current meter that are to be considered in this chapter.

138. Effects of Turbulent Flow.—The term "turbulent" will be used to indicate flow conditions other than the simple filamental motion of the particles of water. Turbulency, therefore, is a quality of flow which will differ in degree for different cases. The conditions of stream flow which are considered ideal for current meter measurements will have a small degree of turbulency and the effect on the behavior of the meter will be so slight that the accuracy of the velocity obtained by the use of the meter may be, within 1 or 2 per cent. On the other hand, conditions which prevail in a tail race near the point of discharge from the turbines will be of the highest degree of turbulency and errors of 15 to 20 per cent in meter measurements may be expected. The measurement of flow at such a site would not ordinarily be considered. Between the conditions of very turbulent flow in the tail race and the smooth flow at the ideal measuring section, there is, however, a wide range of possible conditions where

measurements are made and where turbulency is of such a character as to cause error in the recorded measurements. It is important, therefore, that a clear recognition be had of the effect of turbulence on current meters so that whenever measurements are made under turbulent conditions, they may be correctly appraised. Such a recognition may also act as a deterrent in choosing a measuring section where the flow is very turbulent.

Analysing turbulent flow (See Arts. 13 to 17 for a general discussion of the character of stream flow), we find that it differs from simple filamental flow chiefly in the variableness of the velocity. This variableness is of two sorts, namely, (*a*) that of magnitude and (*b*) that of direction.

Due to the vortices formed in the water, as explained in Chap. III, the motion of the particles of water past a point is constantly being either accelerated or retarded. This variation in velocity is not regular but has short or long periods of pulsation, depending on the degree of perturbation. For ordinary flow conditions all the particles of water acting on the meter may be assumed to be moving with the same velocity and subject to the same variations. In cases where the flow is very turbulent, generally marked by violent eddyings and boiling at the water surface, this assumption is not true, for the meter is subject to the action of many threads of velocity resulting in a condition which defies any exact analysis.

Furthermore, due to the vortex action of the water, the direction of the velocities past a point is constantly changing. Although the current of the stream as a whole is normal to the cross-section, the individual paths of flow of the particles of water may be in an oblique direction or even opposite to the general direction of flow. The direction of the velocity is not necessarily confined to a horizontal plane but may have a vertical component as well.

The effect of these two variations in velocity on the behavior of the meter will determine to a large extent the accuracy of the measurements.

139. Requisites of an Ideal Current Meter.—The variations in magnitude and direction of the velocity impose certain requisites on a meter if it is to be capable of registering the true average normal velocity.

Consider first the effect of the variation in the magnitude of the velocity. The mean velocity of flow during the time covered

by an observation will be the arithmetical mean of all the instantaneous velocities to which the current meter has been exposed. In order for the meter to accurately register this mean velocity, two requirements are necessary.

The first requirement is that the time rate of change in angular velocity of the wheel and the time rate of change in the velocity of the water must be the same. This means that a linear relation must exist between the rate of revolution of the wheel and the velocity of the water.

The second requirement is that the meter must be sensitive to all changes in velocity. The sensitiveness will depend on the friction of the bearings, skin friction, and the moment of inertia of the wheel. The bearing friction is practically negligible at other than low velocities due to the very small torque which it can offer against rotation. The skin friction varies approximately with the square of the velocity. The resisting torque offered by the skin friction depends on the shape of the rotating surfaces and their disposition with respect to the axis of rotation.

The third factor affecting the sensitiveness, the moment of inertia, has a greater proportionate effect than either bearing or skin friction. When a wheel is rotating at a constant speed under the action of a certain velocity of flow it will possess a definite angular momentum. This angular momentum will be equal to the product of the moment of inertia and the angular velocity of the rotor. The sensitiveness of the meter will depend on the readiness of the meter to acquire a different angular velocity to correspond to a different velocity of flow. This readiness will be determined by the magnitude of the moment of inertia of the rotor. If the moment of inertia is large, a slight change in the velocity of flow will be insufficient to alter the speed of the wheel and consequently the wheel will not register the true velocity. Therefore, an ideal meter should have its weight so placed as to make the moment of inertia low.

Coming next to the second variation in velocity, that of direction, we find two possible ways in which the meter may fail to register the true normal velocity. The first is due to the method of suspending the meter by a cable. With this type of suspension, the meter is free to take a direction parallel to the direction of the current. It is aided in this by being provided with a tail. The result of such an arrangement is that the oblique velocity is measured instead of the velocity normal to the cross-section.

Using the measured velocity without any reduction because of obliquity would give an exaggerated discharge. Consequently, such a meter should be provided with a direction indicator so that the angle of obliquity can be read and the normal component computed. The difficulty here, if the flow is quite turbulent, may be that the meter will assume various directions during the period of observation and it would be necessary to use an average angle. This would introduce an approximation.

The second way in which the meter may fail to register the true normal velocity is due to its inability to resolve the oblique velocities into their normal components. Such a meter is not free to swing with the current but has its direction fixed parallel to the normal flow. The surfaces of the meter should be so shaped that neither vertical nor transverse velocities will have any effect on the registration of the wheel. Only the normal component should cause a rotation of the wheel. Such a wheel would have to rotate about a horizontal axis parallel with the normal direction of flow.

From the foregoing, the requisites of a current meter which will give the true average normal velocity may be summarized as follows:

1. Linear rating curve.
2. Low moment of inertia.
3. Fixed in direction.
4. Horizontal axis.
5. Ability to resolve oblique velocities into normal components.

140. Other Requirements for a Current Meter.—Aside from the theoretical requirements of a good current meter, there are other requirements of a practical nature which should not be lost sight of. Such requirements would include the following:

1. The meter should be of simple and compact construction.
2. It should be light in weight but strong enough to withstand rough usage.
3. It should be capable of being dismantled and packed in a box convenient for carrying.
4. The friction of its bearings should be small in order that low velocities may be measured.
5. The bearings should be protected from silt and other grit, be non-corrosive, and subject to as little wear as possible.
6. The rotor should be able to throw off floating debris and should be otherwise protected from damage.

141. Examination of Cup-shaped and Screw-shaped Meters. Since the current meters, in present-day use, are either of the anemometer type or the propeller type, these two types will be examined in the light of the requisites for the ideal meter which were set down in the previous article.

Cup-shaped Meter.—1. Except for velocities below 0.5 ft. per second, the meter has a linear rating curve.

2. The greater part of the mass of the meter is located near the rim of the wheel, thereby making its moment of inertia relatively large for the given mass of material.

3. The axis of the meter is vertical.

4. and 5. This meter is either suspended by a cable or is fastened to a rod. In the first case, its direction is not fixed but depends on the direction of the current. It measures the full value of the velocity. Furthermore, whether its direction is fixed or not, the cups are exposed to the maximum horizontal velocities. Consequently, this type of meter is bound to overregister if there are any appreciable oblique velocities.

The action of the vertical components of velocity would be such as to rotate the cups in a direction counter to the normal direction because of the shape of the surface of the cups and thereby cause an underregistering of the meter.

Screw-shaped Meter.—1. As in the case of the cup-shaped meter, its rating curve is linear above 0.5 ft. per second.

2. The moment of inertia is relatively low, due to the distribution of the mass of the blades.

3. Its axis is horizontal.

4. Some types are held fixed in direction and others are free to swing with the current.

5. The shape of the surfaces of the rotor blades is such as to resolve the oblique velocity into its normal and transverse components, provided the meter is held fixed in direction. If the meter is supported by a cable, and provided with a tail, it will measure the full value of the velocity and if the angle of obliquity is not taken into account, the discharge will be high.

142. Experimental Tests of Behavior of Meters.—The examination of the cup-shaped and the screw-shaped meters, by comparison with certain criteria for an ideal meter, furnishes an idea of what may be expected of existing meters under unfavorable conditions and suggests certain features which are essential in a proper design of a current meter. Such an examination is

rational but entirely theoretical and, to make it more conclusive, actual tests should be made of these meters under turbulent conditions.

Several tests have been made to determine the behavior of different types of meters under velocities varying both in magnitude and direction and also to investigate the errors made by using a still-water rating curve for deducing the actual velocities of streams. In the following articles, some of the tests which have been made will be given.

143. Tests of F. P. Stearns.[1]—In 1883, F. P. Stearns made a series of tests with the Fteley meter to determine the effects of velocities varying in magnitude and direction. The tests were carried out in a conduit over a course of 100 ft. in length. The first series of tests was made by moving the meter over the course at an irregular rate and comparing the number of revolutions with those obtained when the meter was moved with a uniform rate and an equal velocity. The experiments showed that the effect of the irregular movement was to give an excess of discharge although of a small amount. The excess ranged from 0.5 to 5 per cent, averaging about 4 per cent. In two instances, the error was negative, being about −0.5 per cent.

The second series included tests for the effect of obliquity. The results of these tests showed about 0.5 per cent increase for an angle of 7 deg. 48 min., about 2 per cent decrease at 10 deg. 48 min., and a decrease of about 10 per cent at 40 deg. 58 min. Volumetric tests of the effect of irregular flow were carried out by means of a weir which had been accurately calibrated. The velocities ranged from 1.71 to 2.93 ft. per second. The results of these tests showed but small disagreement between the discharge as obtained by the meter and the weir, ranging from 0.2 per cent above to 1 per cent below.

144. Tests of C. F. Rumf.[2]—A series of tests were made by C. F. Rumf in the rating tank of the Renssalaer Polytechnic Institute in 1912, which included an investigation of the behavior of current meters when subjected to cross-currents. A small Price meter of the acoustic type, converted for electric recording, and a Fteley-Stearns meter were used in the investigation. They were turned at various angles from 0 to 90 deg. with the longitudinal axis of the tank. The velocity of flow was 3 ft. per second.

[1] *Trans. Am. Soc. Civ. Eng.*, vol. 12.

[2] *Engineering News*, vol. 71 and *Proc. Eng. Soc. of West Philadelphia*, vol. 30.

Figure 91 shows the results of these experiments. The curves are as follows:

$$R_L = \frac{\text{Rev. per sec. of cup meter at various angles turned to left}}{\text{Rev. per sec. of cup meter at 0 deg.}};$$

$$R_R = \frac{\text{Rev. per sec. of cup meter at various angles turned to right}}{\text{Rev. per sec. of cup meter at 0 deg.}}.$$

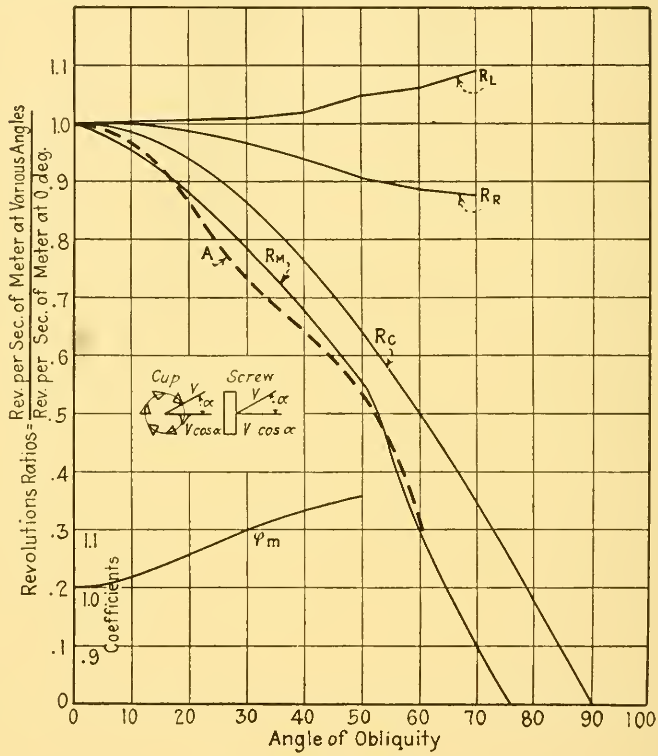


FIG. 91.—Showing the effect of oblique currents on a Price meter and a Fteley meter as observed by Rumpf. Curve A has been added to the diagram and is constructed from the experimental data of Wickham and Worthington (see Art. 147). (*Reproduced from an article by Moody in Proc. Eng. Soc. of W. Pa., Vol.* 30.)

$$R_M = \frac{\text{Rev. per sec. of screw meter at various angles}}{\text{Rev. per sec. of screw meter at 0 deg.}};$$

$$R_C = \frac{\text{Rev. per sec. of meter revolved at various angles}}{\text{Rev. per sec. of meter at 0 deg.}}$$

$$= \frac{R \cos \varphi}{R} = \cos \varphi.$$

$$\varphi m = \frac{R_C}{R_M} = \text{coefficient of screw meter.}$$

The curve, R_C, gives the ratio of the normal component to the diagonal velocity for any angle of obliquity. This would be the curve of the ideal meter.

Examining the results of the test of the Price meter, it is seen that the readings of the meter were different when turned at the same angle but on opposite sides of the normal, being higher when turned to the left than when turned to the right. This difference was attributed to the yoke of the meter. Neither curve approaches the ideal curve, and if their mean is plotted, it would be practically unity, which would indicate that the meter measures approximately the maximum velocity of the stream, regardless of the direction in which it points.

The readings of the screw meter were always lower than the resolved component, which was just the opposite from the results obtained with the cup meter. The general shape of the curve drawn through the points is similar to the shape of the resolved velocity curve. This would seem to show that the design of the Fteley meter is almost ideal for resolving oblique velocities, and could be made more accurate by a slight change in the shape of the blades.

145. Tests of E. H. Brown and F. Nagler.[1]—Experiments were carried out by E. H. Brown and F. Nagler to determine the possible causes of the over-registering of the cup type of meter. For this test, the flow of a 42-in. circular discharge main was used, the water being from 12 to 14 in. deep and having a fairly constant velocity of 4 ft. per second. The meters which were tested were held by a rigid support, capable of being rotated about an axis in such a way that the center of the meter head remained in a fixed position for any angle.

The tests were carried out for two types of Price meters, the large Price and the small Price. Inasmuch as the large Price meter is no longer in general use, only the results of the test of the small Price will be taken up.

The meter was revolved about a horizontal axis and readings were made every 5 deg. up to 90 deg., above and below normal. Also, the meter was revolved about a vertical axis and similar readings were made.

Figure 92 shows the results of the tests. Curve A is drawn through the observed points when the meter was inclined vertically with the axis of the stream. Curve B is the same for the

[1] *Proc. Eng. Soc. of Western Pennsylvania*, vol. 30.

horizontal obliquities. Curve *C* is the 100 per cent circle, curve *D* is the resolved velocity circle, and curve *E* is a theoretical combined curve of *A* and *B* based on probabilities.

As in the test by Mr. Rumf, the meter registered higher values when turned to the left than when turned to the right, measuring

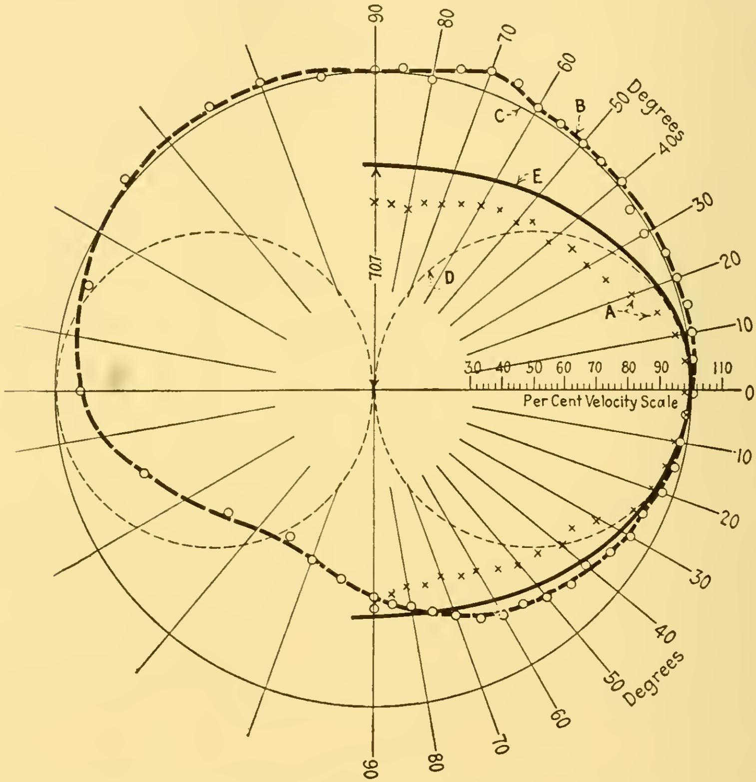


FIG. 92.—Showing effect of oblique currents on the behavior of a small Price meter as determined by Brown and Nagler. Points o—taken to right and left. Points x—taken above and below. (*Reproduced from article by Brown and Nagler, Proc. Eng. Soc. of W. Pa., Vol.* 30.)

slightly greater than 100 per cent when turned to the left. When tilted about a horizontal axis the meter underregistered. To determine the probable net effect of the horizontal and vertical obliquities, if the meter had been subjected to both velocities simultaneously, the theoretical curve *E* was constructed by means of the method of least squares. It will be seen that this curve very closely approximates the resolved velocity curve *D*

as far as 45 deg., which is the limit of obliquity for the majority of conditions.

The approximate coincidence of the curves *D* and *E* led these experimenters to conclude that if the Price meter is supported by a fixed rod in turbulent water where the flow is disturbed in all directions, the registered velocities will be fairly accurate but if the velocity has a decided oblique direction of flow, the meter had better be suspended by a cable and in which case it would register excess velocity.

Their conclusion was that the ideal meter should "register only the true components of velocity when subject to flow from any direction," when supported in a fixed position. Such a meter "must have identical vertical and horizontal characteristic curves." Furthermore, such curves would correspond to the theoretical curve *E*.

146. Tests at Massena, New York.[1]—In 1911, B. F. Groat, while making some efficiency tests on recently installed turbines of the St. Lawrence Power Co. at Massena, N. Y., took occasion to make some comparative tests of a Price meter and a Haskell meter. The current meter measurements were made in tail races where the flow was quite turbulent although not excessively so.

The results of the measurements by the two meters showed that there was considerable difference in readings of the two meters, the Price meter having the higher readings.

These meters were rated at three different times and under three different sets of conditions. From an inspection of the plotted readings, Mr. Groat observed that the ratings of the Haskell meter never varied by more than 1 per cent but that the ratings of the Price meter differed by as much as 5 or 6 per cent up to velocities of 5 ft. per second. From this he concluded that the varying conditions of flow produce a greater effect on the behavior of the cup meter than on the screw meter.

Furthermore, Mr. Groat also observed that the ratings made in other than still water showed a higher reading than that corresponding to the same velocity when read from the maker's rating curve. This led him to suspect that the still water rating curve for a cup meter gave minimum values for the revolutions of a wheel corresponding to any velocity, and that any disturb-

[1] *Trans. Am. Soc. Civ. Eng.*, vol. 76.

ance in the flow would cause the meter to over register, other things being equal. In a similar way, the still water rating curve of a screw meter represented the upper limit, and any disturbance in the flow would decrease the reading of the meter.

The deviations for the Price meter were about six times as large as those of the Fteley meter, but in a contrary sense. From this inspection, Mr. Groat concluded that the true velocity could be obtained by subtracting six-sevenths of the difference of the two readings from that of the Price meter or by adding one-seventh of the difference to the reading of the Haskell meter.

In the measurement of the discharge of the tail races this method was applied.

Some of Mr. Groat's conclusions were the following:

1. When a cup meter is run in perturbed water, it will register a larger number of revolutions per second than a perfect still-water rating would indicate.

2. When a screw meter is run in perturbed water, it will register a smaller number of revolutions per second than a perfect still water rating would indicate.

3. In the foregoing sense, a cup meter is affected relatively to a much greater extent than a screw meter.

4. Either type of meter when run in perturbed water will give uniform records in equal times provided these times are sufficiently long, the flow of the water itself being subject to an established regimen.

5. If both types of meters are used simultaneously in perturbed water, the disparity between the discrepant velocities thus determined by the still-water rating may be taken as a basis for correcting the discrepant velocities.

6. It would seem to follow that current meter observations based on still-water ratings without further correction should be made with great caution. On the other hand it seems certain that the correction for a cup meter when run at a good meter station on an open river is not large, while the corresponding correction for a screw meter may be negligible.

147. Tests Made at the Massachusetts Institute of Technology.[1]—A series of tests were carried out by two students under the direction of members of the staff of the Civil Engineering Department of the Massachusetts Institute of Technology, for the purpose of determining the effect of oblique currents on the recorded velocities.

[1] See *Thesis* of Wickham and Worthington, C. E. Department M. I. T. 1925.

Two meters were used: one a Fteley meter and the other a Price meter. The tests were carried on in a small stream about 20 ft. wide and 3 ft. deep, when the flow was practically uniform, the velocity being about 2¼ ft. per second.

The Fteley meter was turned through horizontal angles up to 60 deg. with the normal to the cross-section and the velocities measured at each 10-deg. point. Curve *A*, which has been drawn on Rumpf's curves, shows in Fig. 91 the results of the tests. It will be seen that the meter resolved the oblique velocities fairly well but gave too low values.

The tests of the Price meter were confined to the effect of tipping the meter in a vertical plane so that there were components of velocity striking the top of the meter. The results of these tests showed the observed velocity was less than the normal velocity.

148. Accuracy of Meters at Low Velocities.—At low velocities, say below 0.5 ft. per second, the current meter should have an even greater accuracy than at higher velocities, because of the greater relative error occurring in a velocity than in a high velocity, for the same inaccuracy in measuring.

If we examine the low-velocity characteristics of current meters, we find that the behavior of the meter is unreliable for velocities below 0.5 ft. per second and may even be unsuited for the accurate measurement of any velocity below 1 ft. per second. This unreliability is due to effect of the friction, slip, and inertia mentioned earlier. These factors determine the velocity at which the wheel commences to turn and also the relation between the speed of the wheel and the velocity of the water. As we have seen, this relation is a curvilinear one, and in cases where there are variations in the magnitude of the velocity this relation would produce considerable error.

Oblique velocities are often to be found in sluggish streams and their effect on the meter which is free to swing will be the same as at higher velocities.

The bearing friction of the Fteley meter makes that meter less sensitive than the Price meter with its single pivotal support.

149. Tests of Fortier and Hoff.[1]—The starting velocities of various makes of meters have been investigated by Fortier and Hoff for the purpose of finding wherein the meters in present-

[1] *Eng. News-Record*, vol. 85.

day use are deficient and of designing a new type of meter suitable for low-velocity measurements.

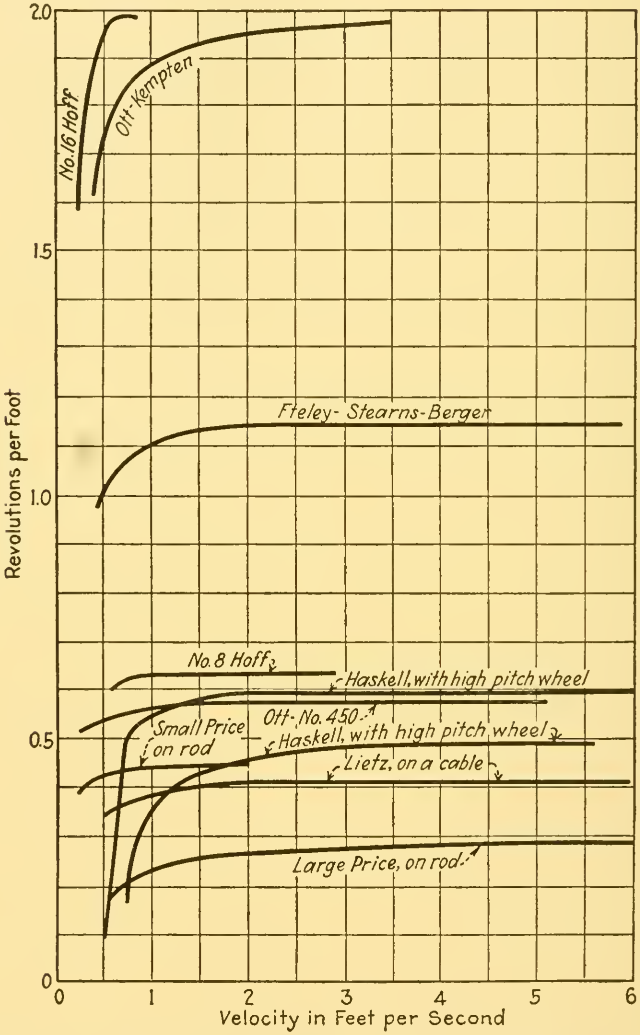


Fig. 93.—Curves showing the starting velocities for various types of meters as affected by inertia. The curve for the ideal meter would be a horizontal line indicating an identical number of revolutions of the meter for all velocities. (*Reproduced from article by Fortier and Hoff in the Engineering News-Record, Vol.* 85.)

The results of their tests are shown, in part, by the curves in Fig. 93. These curves show the starting velocities for several

meters. At some velocity which they called the "critical velocity," the curve becomes straight and remains straight for all higher velocities. The nearer the "critical velocity" approaches zero and the sooner the curve becomes straight, the more sensitive is the meter.

These curves include one drawn for a meter devised by Fortier and Hoff, called the Hoff meter. It is of the propeller or screw type and is designed to resolve the oblique velocities. The axis of the meter support is placed directly above the propeller. The friction of the propeller is kept at a minimum by using a material having a specific gravity equal to that of water.

The tests of this new design gave very favorable results. It will be noticed, however, that the small Price meter showed a fairly flat curve above 0.75 ft. per second.

150. Tests of E. B. H. Wade.[1]—A consideration of low-velocity characteristics should include a resumé of the investigations by E. B. H. Wade, Director of Research of the Ministry of Public Works, Egypt. These investigations were carried on with the view of designing a meter which would have a greater accuracy at low velocities than was obtainable with the current meters then in use.

Briefly, his idea was to design a screw meter whose mechanical system was such that the wheel would be caused to rotate at some constant speed while the velocity of flow was zero and would change its speed when acted upon by flowing water, the change in speed being taken as a measure of the velocity of the water. This, it will be seen, differs from the present type of current meter in which the wheel rotates only when acted upon by the flowing water and the speed of the wheel is taken as a measure of the velocity of the water.

Theoretically, the advantage of this type of meter lies in the removal of the uncertainty of the friction when the meter is barely rotating and in maintaining the same degree of sensitiveness for velocities near zero as for velocities of 1 ft. per second and higher. Tests of this meter have indicated the possibility of measuring velocities of 0.1 ft. per second with an accuracy of 4 per cent, provided the blades have the proper pitch and the driving weight is just the right size.

[1] *Physical Dept. Papers* 6 and 7, Ministry of Public Works, Egypt, Cairo, 1922.

Whether or not such a meter is practical remains to be proven. Its gain in accuracy has been at the sacrifice of simplicity and it is questionable if the gain is worth this sacrifice.

151. Conclusion.—In the foregoing articles of this chapter, the weaknesses of the cup-shaped and the screw-shaped meter have been emphasized. This has been done purposely, in order to impress upon the student the idea that current meters are not fool-proof. Their use requires care, both while being manipulated and while being handled. Furthermore, suitable conditions of flow must be had if accurate work is expected. Where the flow is turbulent, the meter is called upon to do what it was not designed to do. Because of their design, some meters are less adversely affected by turbulent flow than are others.

The limitations of the Price meter have been recognized by the engineers of the U. S. Geological Survey, who use that type exclusively. Consequently, they exercise special care to restrict the use of the meter to measurements where these unfavorable conditions do not exist. The requirements of such a section have been considered in an earlier chapter.

CHAPTER XI

CURRENT METER MEASUREMENTS

152. Structures for Making Measurements.—The structure from which the current meter measurement is made may be either a bridge, a cable, or a boat; and in cases where the stream is shallow, the measurement may be made while wading. The choice will be governed by a consideration of the suitability of the channel section as explained in Chap. IV.

153. Bridge Stations.—An existing bridge may be used if the channel section at the bridge has the requisities of an accurate

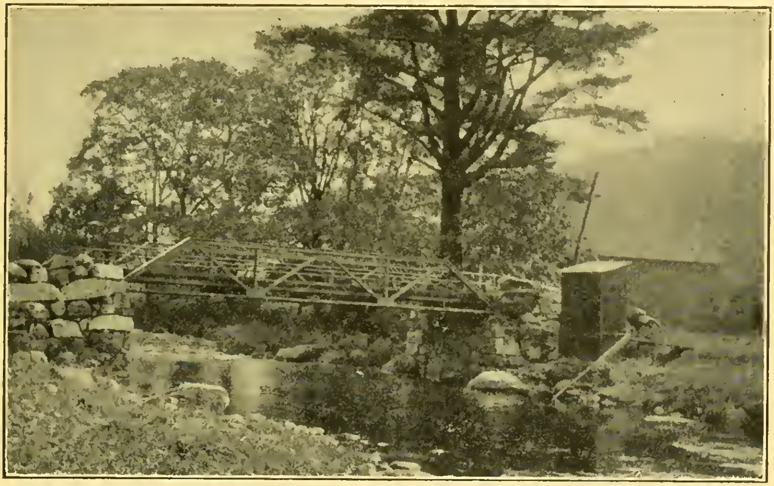


Fig. 94.—U. S. G. S. gaging station, Otter Brook near Keene, N. H. (*Courtesy, U. S. Geological Survey.*)

measuring section. Such a structure is often tempting because it is the most convenient. However, if the channel section is bad, the error in the records obtained will outweigh any apparent convenience. Figure 94 is a highway bridge on Otter Brook, near Keene, N. H., which is used by the U. S. Geological Survey. A good feature of this station is the absence of any pier in the center of the stream to obstruct the flow.

Where there is no existing bridge and the stream is not too wide, a small foot bridge may be constructed over the stream. Such a bridge may be a suspension bridge or it may be made of two or more trussed beams of the queen type. The presence of

FIG. 95.—Chase Mill stream, E. Machias, Me., gaging station of the Massachusetts Institute of Technology.

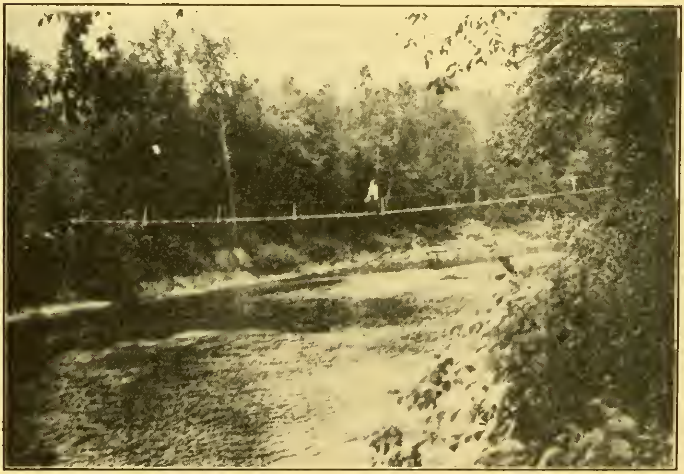


FIG. 96.—Footbridge over the Passumpsic River near St. Johnsbury, Vt. (*Courtesy, U. S. Geological Survey.*)

piers in the center of the stream is undesirable for they cause disturbances in the flow of the stream. Figure 95 is a photograph of one of the bridges used at the gaging station of the Massachusetts Institute of Technology, near Gardner Lake,

East Machias, Maine. Figure 96 shows a suspension bridge over the Passumpsic River at St. Johnsbury, Vt. which is used in measuring. Figure 97 illustrates a type of wooden bridge recommended by the U. S. Geological Survey[1] and may be used on spans up to 80 ft. Bridges should be either creosoted or painted with a weatherproof gray paint.

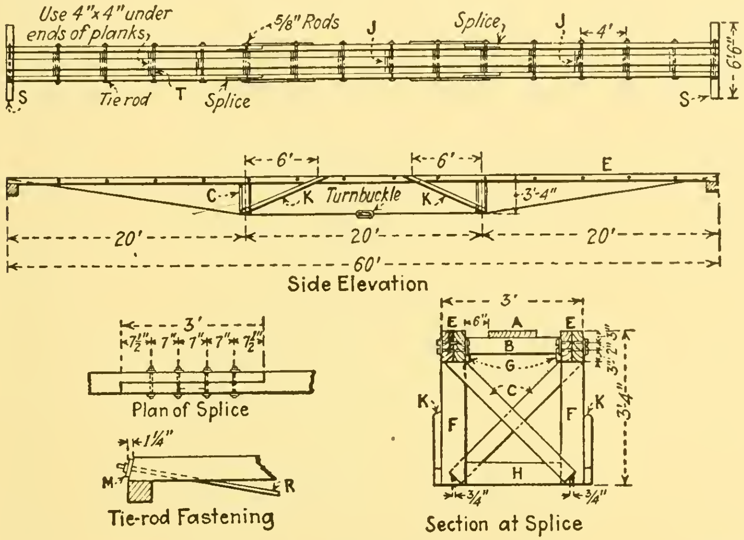


Fig. 97.—Plans for standard footbridge. (*Reproduced from Plate* XXXIII, *U. S. Water Supply Paper* 371.)

154. Cable Stations.—Where no existing bridge is accessible and the stream is too broad for a foot bridge, a cable may be used. The paraphernalia required for a cable station consist of a cable and its accessories, the supports and anchorages for supporting it, and a car for the observer. The U. S. Geological Survey has established a large number of cable stations and has experimented with various designs of supports and cars. It now has certain standard requirements which give safe, durable and economical designs. These have been published in *Water Supply Paper* 371 and much of the following information is from that paper.

The cable, recommended by the U. S. Geological Survey, should be composed of six strands, each strand containing seven wires around a hemp center. The wires should be of the best

[1] *U. S. Water Supply Paper* 371.

quality of galvanized plow-steel. The cable should be free from any splices and should be $\frac{5}{8}$ in. for spans under 400 ft., and $\frac{3}{4}$-in. cable for spans between 400 and 650 ft. The correct amount of sag when the cable is loaded may be determined from computations.

The supports should be of the very best material, free from any defects and thoroughly seasoned. They should be painted with two coats of gray paint as soon as constructed. The sizes of the main posts are determined by the requirements of the load which varies with the length of the span. The main posts are framed with a 1:5 batter. The frames should be supported on mud sills in order to distribute the load effectively.

The anchorages are generally either deadmen, trees, or rock ledges. Deadmen are made of timber or concrete, preferably the latter. If made of timber, they are logs rather than dimension timber. It is desirable that they be creosoted or treated with some other preservative. Charring of the timber will serve to preserve it. In any event, the deadmen should be inspected at least once in four years to ascertain their condition. They should be not less than 12 in. in diameter and 8 ft. long according to the experience of the U. S. Geological Survey. The length of the log and the depth of the burial of the log will depend on the stress in the cable and the nature of the soil. Where the station is a permanent one, the anchor may be of steel or concrete 75-lb. railroad rails incased in concrete.

Where a suitable tree may be found, this will serve very well. The tree should be sound and of sufficient size to withstand any stress placed on it by the cable. Its bark should be protected by some suitable material placed between the cable and the tree. The cable should be secured so that it will not slip up the tree.

Where there is available only a rock ledge for an anchorage, a hole may be drilled in the ledge at least 2 ft. deep and 2 in. in diameter and a $1\frac{1}{2}$-in. steel bolt having one end provided with a slit and the other provided with a steel wedge driven into the hole. The steel wedge expands the bolt as it is driven into the hole. The space around the bolt is filled with cement, grout, sulphur, Babbit metal, or brimstone.

The car is a rectangular-shaped box without any top or bottom. It is approximately 2 by 6 ft. in cross-section and 1 ft. deep. It is carried on the cable by two galvanized hangers which pass under the car, and which are fastened to pulleys on one end.

Figure 98 shows a support and cable car used by the U. S. Geological Survey at its station at Charlemont, Mass., on the Deerfield River. Figure 99 shows a general view of the station at Charlemont.

155. Boat Stations.—Boat stations are unsatisfactory because of the difficulty of keeping the boat steady during a measurement. The meter should be kept clear of any effect due to the boat on the motion of the water. This may be done by suspending the

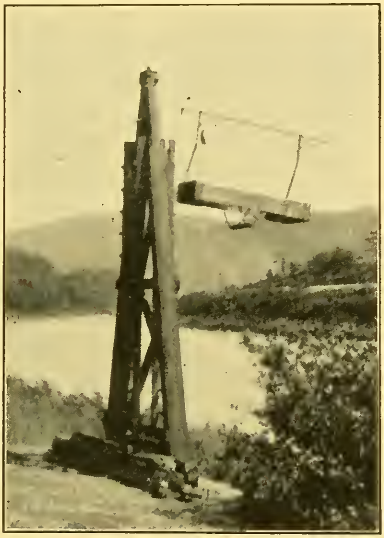


FIG. 98.—Cable car and support. (*Courtesy, U. S. Geological Survey.*)

meter from a pole hooked on the cable, and then extending the pole and meter at a proper distance from the boat. The methods for keeping the boat on range or for locating the position of the boat would be the same as in the case of sounding from a boat, considered earlier.

Measurements from boats are rarely necessary and it is the exception rather than the rule to find a case where such measurements are made.

156. Wading Stations.—Wherever the depth of the water and the velocity of the current are such that the observer can reach all parts of the section, wading measurements may be made. The observer should stand below the rope or tape stretched across the stream and at one side of the meter in order not to interfere with the flow of the water. It is obvious that standing in front

or in the rear of the meter will disturb the flow of the water and the action of the meter. Figure 100 is a view of a wading measurement being made on the Waiahole Stream, Oahu, Hawaii.

Fig. 99.—View of U. S. G. S. gaging station on the Deerfield River at Charlemont, Mass. (*Courtesy, U. S. Geological Survey.*)

Fig. 100.—Velocity-area gaging station, Waiahole Stream, Oahu, Hawaii. Measurement by wading. (*From U. S. Water Supply Paper.*)

157. Spacing of Verticals.—The general procedure in a current meter measurement is to divide the cross-section into a number of relatively narrow vertical strips and find the mean velocity

for each of these partial areas. The summation of the products of the partial areas and their corresponding mean velocities will be the discharge for the whole cross-section.

The first step to be taken in the measurement is to divide the cross-section into an appropriate number of strips. Each strip, or partial area, will be bounded by the water surface, the contour of the bed, and two imaginary vertical lines known as "verticals." The velocities to be obtained will be the mean velocities in these verticals.

The spacing of these verticals will be governed partly by the shape of the profile of the bed and partly by the conditions affecting the velocities. For a stream of tolerably smooth bed and having evenly distributed velocities, the verticals may be spaced at equal intervals. Where the contour of the bed is irregular, it will be necessary to space the verticals in such a way as to obtain an accurate profile of the bed, as was explained in Chap. VI. In this case the spacing would not be regular but would be made to correspond to the points where the breaks in the profile occurred.

The transverse variation in velocities should also be considered in determining the position of the verticals. Near vertical banks, where the velocities decrease rather sharply, or in the vicinity of any obstruction such as a pier or a rock, where the currents are somewhat disturbed, the verticals should be spaced near enough together to allow the measurement of these velocities.

Although it is generally advisable to have at least twenty verticals in a cross-section, in some cases this number may be decreased. Where the profile is smooth the verticals may be spaced as follows:

Stream Width	Spacing of Verticals
5 to 10 ft.	½ to 1 ft. apart.
10 to 30 ft.	1 to 3 ft. apart.
30 to 100 ft.	3 to 5 ft. apart.
Over 100 ft.	5 to 20 ft. apart.

After the spacing has been decided upon, the points should be marked on the structure from which the measurements are to be made. It is desirable to have a permanent reference point from which to start marking. Where successive measurements are to be made from time to time, it is necessary that the same points of measurement be readily obtained. The ready location of these verticals will be facilitated if the points are

marked permanently on the floor or rail, if a bridge station, or on the cable, if a cable station. In the case of boat or wading measurements, the points may be located as described in Chap. VI.

158. Measurements of Depths.—The measurement of the depths in the verticals may be carried out prior to any velocities being measured, or it may be done at the time the velocities are measured in each successive vertical. In the first case, a sounding rod or a cable may be used, as described in Chap. VI. In the second case, the meter cord, having the meter and weight attached, may be used. When using the meter cord, the best procedure to follow is to lower the meter and weight to the bed and, with the cord held taut, mark a point on the cord opposite some reference mark on the structure; then raise the meter and weight until the bottom of the weight (assuming it to be at the lower end of the hanger bar) is in the water surface, and mark another point on the cord opposite the reference mark on the structure. The distance between the two marks may be measured with a tape to determine the depth. Actually, there is no necessity to place a mark on the cord where the depth to be measured is not great. The ring, or zero point, of a metallic tape may be held at the first point with the fingers of one hand and the cord and tape run through the other hand until the weight is in the water surface and the second point read directly from the tape.

In using the meter cord with the meter and weight attached, care should be taken to see that the weight is resting on the bottom in such a way as to give the true depth in the vertical. Irregularities in the bed, or small stones, may prevent the weight's resting on the bottom and, in shallow streams, a considerable error in the depth may result. In such cases, it would be better to use a rod.

159. Measurement of Velocity.—The mean velocity in each vertical is most commonly obtained by one of the following methods:

1. Point or vertical-velocity curve method.
2. Two-and-eight-tenths method.
3. Six-tenths method.
4. Integration method.

In methods 1, 2, and 3, the meter is held at the point in the vertical where the velocity is to be measured. Care should be exercised to make sure that the center of the meter is placed

exactly at the intended point. This is most readily accomplished by first suspending the meter so that its center is in the water surface and marking or noting the point on the cord opposite a reference mark on the structure. The required depth can then be measured on the cord and the meter lowered until this mark is opposite the reference mark where it is secured.

The timing of the revolutions of the wheel may be done in either one of two ways. The number of revolutions the wheel turns in a given time may be observed or the time required for a given number of revolutions may be observed. The second method is the more accurate one because the time for a certain number of revolutions may be obtained with a stop-watch to the nearest two-fifths of a second whereas the meter does not record fractional parts of a revolution. Even if an ordinary watch is used in timing, being read to the nearest second only, this second method would be more accurate.

If the second method is used, the rating table should be constructed to give the velocity corresponding to total revolution in total time, as explained in Chap. IX. If only the velocities corresponding to multiples of 10 revolutions are given in the table, the observed number of revolutions should also be in multiples of 10, otherwise it will be necessary to interpolate.

Where the revolutions are indicated by means of a buzzer, the timing should be done with reference to the beginning or end of the buzz.

The required length of time for an observation is between 40 and 70 sec. The U. S. Geological Survey has found by experience that if a sufficient number of verticals is taken the discharge obtained by making the duration of the velocity observations of the above length is generally accurate to within 2 per cent. The rating table used by the Survey covers only this range in time.

160. Point or Vertical Velocity-curve Method.—In the point or velocity-curve method, the measurements are made just below the surface, say at 0.5 ft., and at points located at every tenth of the depth, including one as near to the bed as possible. The number of points may be less than ten if a smaller number can be trusted to define the vertical velocity curve. These measured velocities can then be plotted as abscissae against the corresponding depths as ordinates and a curve drawn through the points. The area is bounded by the vertical, surface, and bed velocities

and the velocity curve can be obtained by planimetering or by applying Simpson's rule, or by any other suitable method. The quotient obtained by dividing the area by the depth will be the mean velocity in the vertical.

This method is probably the most accurate one but it requires a long time to make complete measurements across the section and is therefore not as practicable as a shorter method having a reasonable degree of accuracy.

161. Two-and-eight-tenths Method.—In the two-and-eight-tenths method, the meter is held at two-tenths of the depth and at eight-tenths of the depth and the velocities obtained at these two depths. The mean velocity for the vertical will be the average of these two velocities as explained in Chap. III.

162. Six-tenths Method.—In the six-tenths method, the meter is held at six-tenths the depth measured from the surface, and the velocity so obtained is taken as the mean velocity in the vertical. The accuracy of this method was discussed in Chap. III.

163. Integration Method.—In the integration method, the meter is lowered in the vertical to the bed of the stream and then raised to the surface. This lowering and raising of the meter should be done at a uniform rate, care being taken not to hold the meter longer at the bottom of the vertical than elsewhere. The total integration of the meter constitutes a measurement. During this passage of the meter the total number of revolutions and the time elapsing are noted. With these data the average number of revolutions per second can be found, and then, by using a rating curve, the velocity in feet per second can be obtained. This velocity will be considered the mean velocity in the vertical since all the velocities in the vertical are supposed to have acted equally upon the meter.

This method is, theoretically, an accurate one but manipulation of the meter has much to do with the accuracy. Hence, the amount of reliance to be placed upon the velocity so determined will depend upon the ability and experience of the observer. Murphy, in his Cornell University experiments, observed fluctuations of from +0.6 to −3.0 per cent in the ratios of the differences between the velocities obtained by the point method and the integration method, to the velocities obtained by the point method.

Stearns[1] found that the effect of integration with a Fteley meter at too rapid a rate was to diminish the meter measurement. Although the most rapid rate admissible for obtaining accurate results was somewhat indefinite, he concluded that the rate should not exceed five per cent of the velocity of the current.

The cup-shaped meter is not suited for integration work because, as was pointed out earlier, the vertical velocities act on the cups in such a way as to turn them in a direction which is counter to that resulting from the action of the horizontal velocities. The Fteley meter blades are surrounded by a band which serves to prevent any rotation of the wheel due to vertical velocities acting on the blades. Wheels of the Haskell type are also but slightly affected by the vertical movement of the meter.

Additional causes of error in an integration measurement are found in the inability of the meter to measure the bed velocities because of its construction and in the irregular movement of the meter through the water when being lowered and raised. The first would result in the omission of slow velocities from the integration and the second would give greater weight to some velocities than to others.

164. Integration of the Section as a Whole.—In this method the meter is carried across the stream in a zigzag course from top to bottom, the number of revolutions and total time being observed as before, and the velocity thus obtained is taken as the mean velocity in the section.

Referring to the Cornell University experiments, it was found that the results did not give a well-defined line. The ratios of the differences between the velocities found by weir measurement and integration method to the velocities found by weir measurement varied from +5.35 per cent to −39.30 per cent, the velocity in the latter case as determined by weir being 0.229 ft. per second.

165. Advantages of Current Meter Measurements.—The following advantages of the current meter should be noted.

1. Observations are taken in one section only.
2. Observations are taken more rapidly than with floats.
3. A smaller party is required.
4. Velocity variations are averaged during the time of an observation.
5. Velocities close to bed and banks of the stream can be obtained.

[1] *Trans. Am. Soc. Civ. Eng.*, vol. 12.

6. Measurements may be made rapidly.
7. One man can make the measurement if necessary.

166. Discharge Measurement Notes.—The discharge measurement notes should include (*a*) sufficiently complete data to enable the calculation of the discharge, and (*b*) other data describing the conditions under which the measurements were made.

DEPARTMENT OF THE INTERIOR
UNITED STATES GEOLOGICAL SURVEY
WATER RESOURCES BRANCH

DISCHARGE MEASUREMENT NOTES

File No.
Washington..........
Field..........
No. of Meas..........

Date.............. 192

.... River, at ....

Dist. from initial point	Depth	Depth of observation	Revolutions	Time in seconds	Velocity: At point	Velocity: Mean in vertical	Velocity: Mean in section	Area	Mean depth	Width	Discharge
......	......	......	......	......	......	......	......	......	......	......	......
......	......	......	......	......	......	......	......	......	......	......	......
......	......	......	......	......	......	......	......	......	......	......	......
......	......	......	......	......	......	......	......	......	......	......	......
Totals	......	......	......	......	......	......	......	......	......	......	......

No.......... of.......... Sheets. Comp. by.......................... Chk. by..........................

Fig. 101.—Form used by the U. S. Geological Survey for keeping discharge measurement notes.

The data necessary to enable the calculation of discharge would be the following:

1. Horizontal distance of vertical from initial point.
2. Depth of vertical.
3. Depth in vertical at which observations are made.
4. Number of revolutions of the meter wheel.
5. Time covered by the observation.

Other data which describe the condition under which the measurements are made would be the following:

1. Date.
2. Name of stream.
3. Location of station.
4. Number of measurement, if one of a series.
5. Names of members of party.

DEPARTMENT OF THE INTERIOR

UNITED STATES GEOLOGICAL SURVEY

MISCEL. MEAS. — WATER RESOURCES BRANCH — REGULAR STATION

DISCHARGE MEASUREMENT NOTES

Date ...., 192 No. of Meas..............

..............................River at / Creek near.................., State of..................

Width .... Area.......... Mean Vel.......... Cor. M. G. H..........

Party .... Disch..................

Staff gage, checked with level and found..

Chain length, checked with steel tape, 12-lb. pull, found ....ft.

" " changed to..........ft. at..........o'clock. Correct length..........ft.

" " corrected on basis of levels to..............ft. at..............o'clock.

Gage reading	Time	Station
....	....	....
....	....	....
....	....	....
....	....	....
....	....	....
....	....	....

Meter No ....

Date rated ....

Meas. began..........; ended..........

Time of meas. (hrs.)...... Method..........

No. meas. sec's.......... Coef..........

Av. width sec.......... Av. depth......

Weighted mean G. Ht ....ft.

G. Ht. change (total)...............

Correct " " " ....ft.

......per cent diff. by..........rating table.

Meas. from cable, bridge, boat, wading. Meas. at..............ft. above below gage.

If not at regular section note location and conditions..............................

.... Area from soundings (date)..............

Method of suspension.......... Stay wire.......... Approx. dist to W S..........

Arrangement of weights and meter; top hole......; middle hole......; bottom hole......

Gage inspected, found.................. Cable inspected, found..................

Distance apart of measuring points verified with steel tape and found ....

Wind.............upstr., downstr., across. Angle of current..........................

Observer seen.................. G. Ht. book inspected ....

Examine station locality and report any abnormal conditions which might change relation of G. Ht. to disch., e. g., change of control; ice or débris on control; back water from; condition of station equipment..

....

Sheet No. 1 of......sheets. If insufficient space, use back of sheet, with reference letters.

June 1918.

FIG. 102.—Form used by the U. S. Geological Survey for making a record of miscellaneous measurements.

6. Mean gage height. (The gage should be read at intervals during the measurement to determine if any change occurs. From these readings the mean gage height can be determined.)

7. Name and number of meter.

8. Time of start and finish of measurement.

9. Measurement method.

10. Information regarding ice conditions, presence of floating debris, direction of wind, angle of current, etc.

Figures 101 and 102 are forms used by the U. S. Geological Survey in the collection of discharge data. In Fig. 101,

Party No.________

STREAM GAGING

Date__________________________ Watch No.__________

Locality__________________________ Watch registered__________seconds in 600.

Meter__________________ No.________ Measurement Method__________________

Sta.	Depth	Meter Depth	Counter Start	Counter Finish	Total Revs.	Time		R. p. s.	Gauge

FIG. 103.—Form for keeping discharge measurement notes where the revolutions of the wheel are read on dials.

columns are provided for the computation of the mean velocity, area, and discharge of the stream. In Fig. 102, space is given for the data necessary to describe the conditions of the measurement.

On another form (not shown) provision is made for supplementary discharge measurement notes. Such notes cover the following points: (*a*) gage, (*b*) observer, (*c*) bench mark, (*d*) gage height corrections, (*e*) measuring equipment, (*f*) channel control, and "zero flow," (*g*) rating, backwater, (*h*) diversions, (*i*) records, (*j*) general, value of records, comparison with other stations, cooperation, etc.

Figure 103 shows a convenient arrangement of headings of columns for use with the Fteley meter where the revolutions are recorded on dials.

167. Formulas for Calculation of Discharge.—The discharge may be calculated from the observed data either by means of a formula or graphically. The former method is the simpler of the two and is more commonly used. Such a formula is a summation of the several partial discharges, these partial discharges being calculated from the observed values of depths and mean velocities in the verticals and the widths of the strips between the verticals.

Several formulas have been used to calculate the discharge, differing in the form of the term expressing the partial discharge. These formulas can be divided into two groups, namely, (*a*) rectilinear and (*b*) curvilinear. In the rectilinear formula, the ordinates are considered in groups of two's or three's and the transverse velocity curve and the bed assumed to be each a part of the perimeter of a polygon. In the curvilinear formula, the ordinates are considered in groups of three's, being combined in a different manner for each formula, and the transverse velocity curve and the bed are each assumed to be parabolic arcs.

J. C. Stevens[1] investigated the rectilinear and curvilinear formulas which were in use and compared the results obtained by their use. From this study he concluded that the curvilinear formulas are only justified when the bed of the stream and the transverse velocity curve are continually concave to the axis of reference. This is seldom true of these curves since the bed is nearly always irregular in contour and some portions of the transverse velocity curve will be convex to the axis lying in the water surface.

The formulas, rectilinear in form, which he found well adapted to conditions are:

$$Q = b_1\frac{(d_0 + d_1)}{2} \times \frac{(v_0 + v_1)}{2} + b_2\frac{(d_1 + d_2)}{2} \times \frac{(v_1 + v_2)}{2} + \dots \; b_n\frac{(d_{n-1} + d_n)}{2} \times \frac{(v_{n-1} + v_n)}{2} \quad (1)$$

and

$$Q = b_1\left(\frac{d_1v_1}{2}\right) + b_2d_2v_2 + b_3d_3v_3 + \dots \; b_n\left(\frac{d_nv_n}{2}\right) \quad (2)$$

[1] *Eng. News*, June 25, 1908.

where $b_1, b_2, b_3, \ldots b_n$ are the spaces between the verticals; $d_0, d_1, d_2, \ldots d_n$ are the depths at the respective measuring points, $a_0, a_1, a_2 \ldots a_n$; and
$v_0, v_1, v_2, \ldots v_n$ the velocities at the corresponding measuring points (Fig. 104).

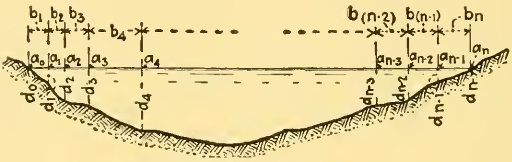


FIG. 104.

168. Graphical Method of Calculating Discharge.—The use of a formula for calculating the discharge directly from the observed data does not discriminate between data which are good and those which are bad. Unless a measurement is extremely bad, so that the magnitude of the velocity or the sounding is quite evidently at variance with the other measurements, it will be included in the computations. With the graphical method the measurements which are in error will be easily detected when plotted and, for this reason, the graphical method has a practical advantage. However, it requires considerable time to make such a determination of the discharge and in most cases of measurement the expenditure of so much time would not be warranted.

The method consists in plotting from a common reference axis the mean velocities and soundings measured in the selected verticals. A smooth curve is drawn through the plotted velocities and is known as the transverse velocity curve. If the velocities lie on a smooth curve, the curve may be drawn through the points, but if the position of the plotted velocities would cause a saw-tooth type of curve to be drawn through them, an average curve is probably better representative of the variation in velocity across the stream.

The profile of the section will be determined by the plotted soundings.

After the two curves are drawn the mean velocity and mean depth for each partial area, say 1 or 2 ft. in width, can be scaled from the plot. The corresponding velocity and depth when multiplied together will give the discharge through an area 1 ft. wide and when multiplied by the width of the partial area will

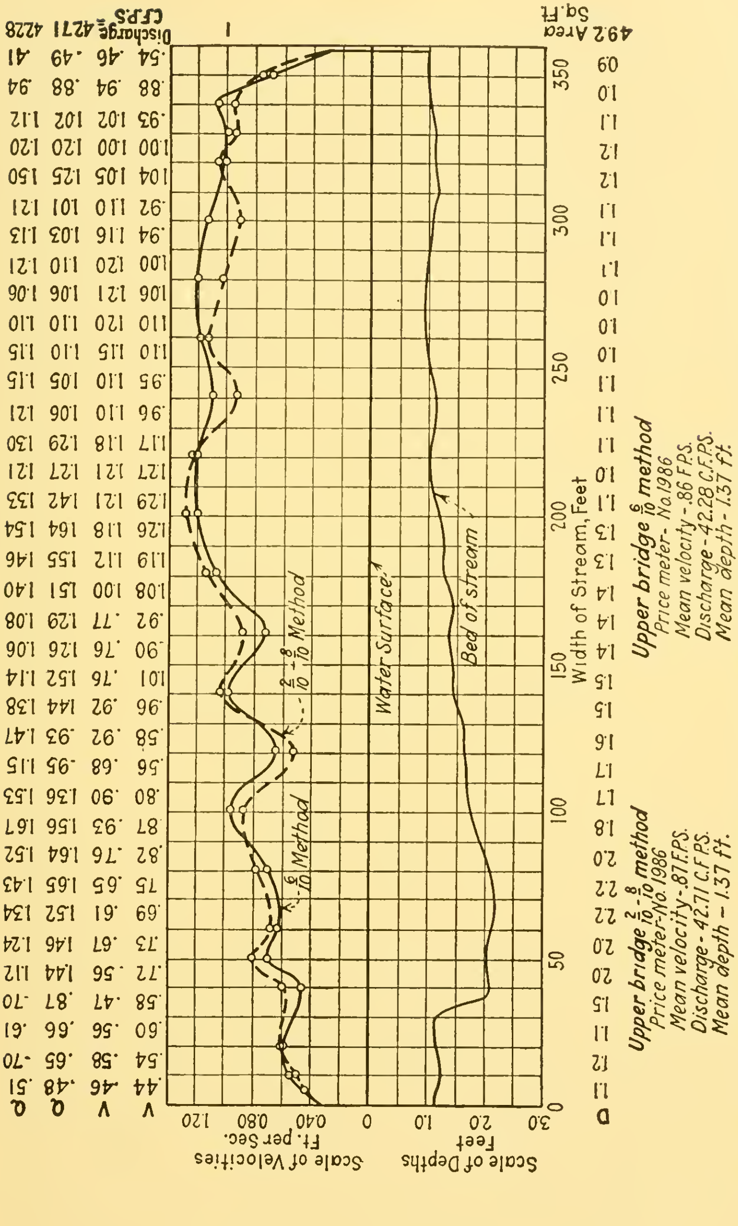


FIG. 105.—Stream gaging measurements, M. I. T. gaging station, Chase Mill Stream, E. Machias, Maine. Illustrating the graphical method of working up discharge measurements.

give the discharge for that partial area. The summation of these products will give the total discharge.

Figure 105 illustrates this method of calculating the discharge graphically. It shows the results of two sets of measurements obtained at the same section but by different methods. The measurements were made by two groups of students of the Massachusetts Institute of Technology at the Institute gaging station connected with its Civil Engineering Camp at East Machias, Me. The profile of the bed was plotted and, with the water surface as an axis, the transverse velocity curves were drawn through their respective points. The section was divided into 1-ft. strips and the depth at the midpoint of each strip was read and entered in the

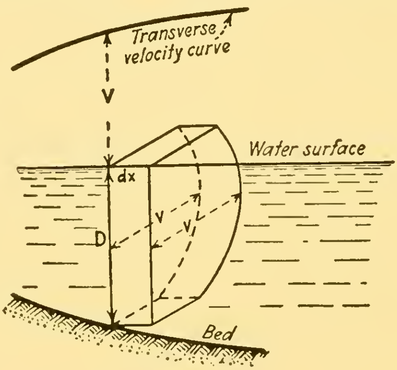


FIG. 106.

horizontal column below the bed profile. The product of each mean depth and the corresponding width of the strip is the partial area. The summation of these partial areas is the area of the section. In this case, since the strips are 1 ft. wide, the summation of the depths is equal to the area. Depths for strips less than 1 ft. wide should be weighted to conform to 1-ft. strips.

Similarly, the velocities at the mid-points of the strips were read from the transverse velocity curves and entered in horizontal columns above the curves. Each velocity was then multiplied by the corresponding depth and the products entered in horizontal columns above the velocity columns. The summation of these partial discharges was the discharge for the entire section.

169. Harlacher's Graphical Method of Computing Discharge. If the discharge be represented by a solid, parallel planes dx distance apart will cut out a slice which will appear as shown in

Fig. 106. It will have plane faces on five sides and the sixth will be curved.

If D is the depth in any vertical and V the mean velocity past that vertical, the differential discharge for a partial area, dx wide, will be $(V \cdot D)\,dx$.

This may be considered to be the product of two quantities, $(V \cdot D)$ and dx. If a curve were plotted, having the widths for abscissae and the value of $(V \cdot D)$ for ordinates, the area underneath the curve would be the discharge for the section. In other words, we might write

$$dQ = (V \cdot D)dx,$$

and, for the entire section,

$$Q = \int_O^W (V \cdot D)dx,$$

the integration extending from one bank to the other.

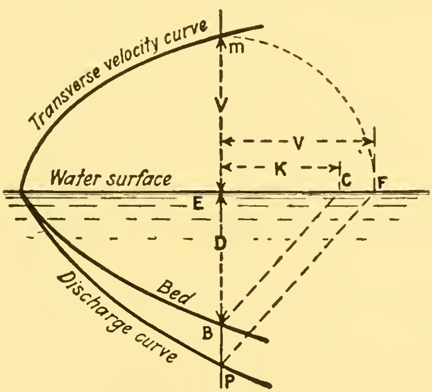


Fig. 107.

Harlacher[1] developed a method for the construction of such a curve which is explained as follows: In Fig. 107 let OM represent the transverse velocity curve, and OB the bed of the stream. For any vertical the velocity will be V and the depth will be D. Construction: choose any distance called K, and lay it off on the axis of reference measuring from the intersection of the vertical with the reference line at E. This distance, K, is read and laid off as so many inches, fractions of a foot, centimetres, etc., and no regard is paid to the scales of the diagram. The size of K will

[1] "Die Messungen in der Elbe und Donan und die Hychometrischen Apparate und Metohden des Verfassers," Leipzig, 1887.

depend on the degree of flatness desired for the so-called "discharge curve."

After laying off the distance, K, the velocity, V, may be laid off from the vertical by swinging an arc of radius V. Then $EC = K$ and $EF = V$. Next draw a line from F parallel to a line joining C and B until it intersects the vertical MEB, produced if necessary at the point P. In the similar triangles, ECB and EFP, the following ratios may be written:

$$\frac{EC}{EB} = \frac{EF}{EP}$$

or

$$\frac{K}{D} = \frac{V}{(EP)},$$

from which $K(EP) = (V \cdot D)$.

Replacing $(V \cdot D)$ by $K(EP)$ in the equation

$$Q = \int_O^W (V \cdot D)dx,$$

we have

$$Q = K \int_O^W (EP)dx.$$

This is in the form desired, having a curve whose abscissae are x and whose ordinates are (EP) corresponding to whatever vertical is used. The integral $\int_O^W (EP)dx$ is the area underneath the curve and is denoted by A.

The discharge, Q, will then be

$$Q = KAS_vS_hS_d,$$

where S_v = scale of velocities.
S_h = scale of widths.
S_d = scale of depths.
K = distance in units used.
A = area under curve in square units measured.

For example, let S_v be 1 in. = 2 ft. per second.
S_h be 1 in. = 4 ft. per second.
S_d be 1 in. = 1 ft. per second.
K = 5 in.
A = 50 sq. in.

Then $Q = 5 \times 50 \times 2 \times 4 \times 1 = 2000$ cu. ft. per second.

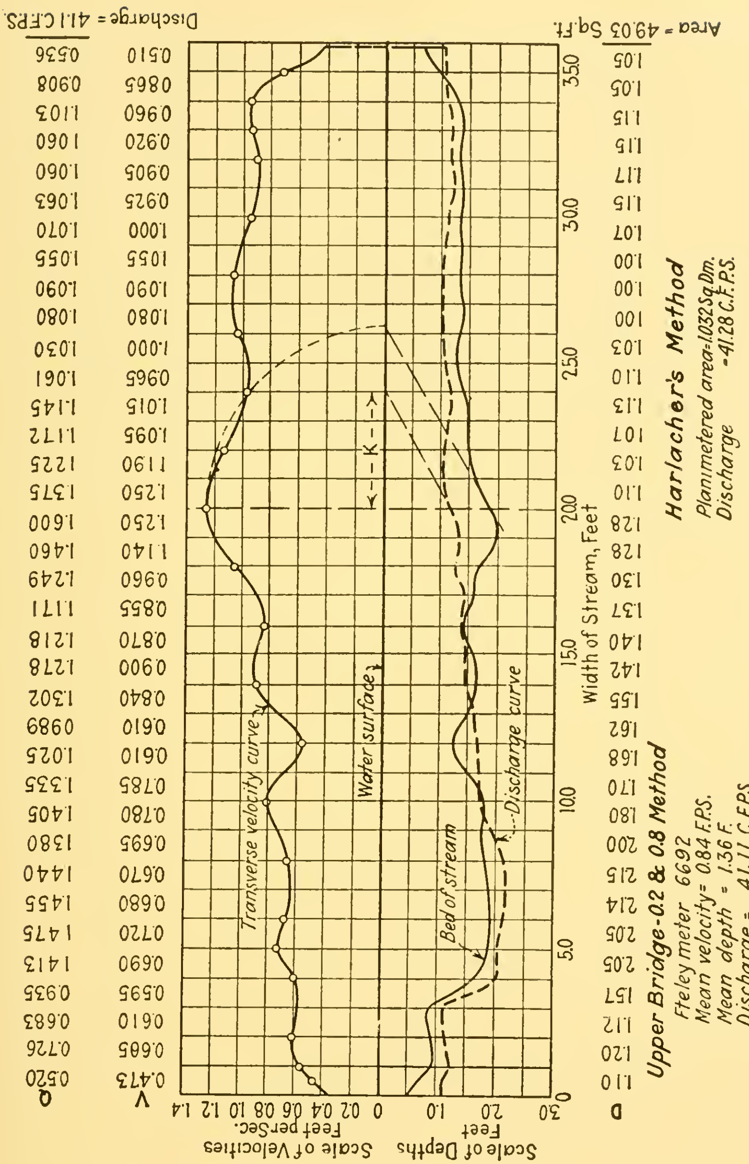


FIG. 108.—Stream gaging measurements, M. I. T. gaging station, Chase Mill Stream, E. Machias, Maine. Illustrating Harlacher's method of working up discharge measurements. The results of Harlacher's method and the ordinary graphical method are compared.

This method is a lengthy one and it is not necessary to construct such a diagram to compute the discharge. Its chief interest is its mathematical exactness.

Figure 108 illustrates the application of this method. The profile of the bed and the transverse velocity curve were plotted. The scales used were: velocity, 1 cm. = 2 ft. per sec.; horizontal, 1 cm. = 1 ft.; depths, 1 cm. = 0.5 ft. K was chosen as 4 cm. Points on the "discharge curve" were located every 2 ft. across the section by the method described above. The "discharge" curve was then drawn through the plotted points and the area bounded by the water surface and the "discharge curve" was planimetered. Its area was 103.2 sq. cm. The discharge was $Q = 4 \times 103.2 \times 0.2 \times 1.0 \times 0.5 = 41.28$ cu. ft. per second.

For comparison, the discharge was also determined graphically as explained earlier and found to be 41.11 cu. ft. per second.

CHAPTER XII

SLOPE AND WEIR MEASUREMENTS

SLOPE MEASUREMENTS

170. Method.—The slope method of measuring the discharge of a stream consists in determining (*a*) the mean area of the channel cross-section, (*b*) the mean hydraulic radius, (*c*) the slope of the water surface, and (*d*) the character of the channel lining, in order to choose a suitable roughness factor. With these data, the mean velocity of the stream may be found by the Chezy formula, $V = C\sqrt{RS}$. The discharge will be the product of this velocity and the mean area of the cross-section. As explained in Art. 19, C is a coefficient depending on the roughness of the lining, the hydraulic radius and, to a slight extent, the surface slope. R is the hydraulic radius and is equal to the ratio of the mean area of the cross-section to the mean wetted perimeter. S is the slope of the water surface.

171. Determination of Slope.—In order to determine the slope of the surface of the stream, it is necessary that a course be chosen that is fairly straight and at least 200 ft. long. The length of the course should be as great as possible, say up to 1000 ft., in order that the per cent error in the determination of the slope shall be as small as possible. This is particularly true for streams of very flat slopes. On the other hand, since the slope should be fairly uniform, it may be necessary to shorten the length of the course in order not to include rapids or other abrupt falls.

The slope will be determined by means of gages placed at the ends of the course and read simultaneously. It is desirable that there be at least two gages at each end, one at each bank, and the average of the two readings be used. Where it is feasible, a gage may be placed near the center of the stream. If placed next to a bridge pier, allowance must be made for disturbances caused by the piers. All the gages should be set with reference to some bench mark and tied together by means of levels. Inasmuch as the gages should be read to hundredths, they should be

protected from all wave action. This is most easily accomplished by surrounding each gage with a stilling box.

Where gages are not installed, the slope may be determined by means of reference marks on posts, trees, bridge piers, etc., these marks being referred to a common elevation.

172. Determination of Area and Wetted Perimeter.—Unless the channel cross-section is uniform it will be necessary to determine the area and wetted perimeter of several cross-sections along the course and use the mean of these areas and wetted perimeters in computing the hydraulic radius. In artificial channels, the section is practically constant so that only one section has to be measured, but in the case of natural streams the cross-sections are different at every point in the stream so that there is no single section which can be used as a basis for the area and wetted perimeter in applying the Chezy formula.

The number of sections to be measured will depend on the length of the course and the configuration of the channel of the stream. Enough sections should be taken to furnish a reasonably approximate value of the average area and wetted perimeter.

173. Coefficient C.—The remaining factor to be determined is C. There are several formulas available for determining its value. One of the best known and one which has had extensive use in the United States is that of Ganguillet and Kutter, which was published in 1869. It is

$$C = \frac{41.65 + \dfrac{0.00281}{S} + \dfrac{1.811}{n}}{1 + \dfrac{n}{\sqrt{R}}\left[41.65 + \dfrac{0.00281}{S}\right]}$$

where n = roughness factor, R = the hydraulic radius, and S = the slope.

A second formula, Manning's formula, published in 1890, is

$$C = \frac{1.486}{n} R^{1/6}.$$

Here n and R have the same meaning as above.

When substituted in the Chezy formula there results,

$$V = \frac{1.486}{n} R^{2/3} S^{1/2}$$

A third formula is that of Bazin, published in 1897, which is as follows:

$$C = \frac{157.6}{1 + \frac{m}{\sqrt{R}}}.$$

m = roughness factor.
R = hydraulic radius.

Any of these formulas will probably give results which are equally satisfactory and the choice of a formula will be governed by convenience. There are also other formulas which have equal merit, and a discussion of these, together with those here given, may be found in any standard textbook on hydraulics.

Values of n in the Kutter formula for various types of channels, as prepared by Horton[1] from an examination of the best available experiments, are given in the following table:

TABLE X.—HORTON'S VALUES OF n. TO BE USED WITH KUTTER'S AND MANNING'S FORMULAS

Surface	Best	Good	Fair	Bad
Canals and ditches:				
Earth, straight and uniform	0.017	0.020	0.0225[1]	0.025
Rock cuts, smooth and uniform	0.025	0.030	0.033[1]	0.035
Rock cuts, jagged and irregular	0.035	0.040	0.045	
Winding sluggish canals	0.0225	0.025[1]	0.0275	0.030
Dredged earth channels	0.025	0.0275[1]	0.030	0.033
Canals with rough, stony beds, weeds on earth banks	0.025	0.030	0.035[1]	0.040
Earth bottom, rubble sides	0.028	0.030[1]	0.033[1]	0.035
Natural stream channels:				
1. Clean, straight bank, full stage, no rifts or deep pools	0.025	0.0275	0.030	0.033
2. Same as (1) but some weeds and stones	0.030	0.033	0.035	0.040
3. Winding, some pools and shoals, clean	0.033	0.035	0.040	0.045
4. Same as (3), lower stages, more ineffective slopes and sections	0.040	0.045	0.050	0.055
5. Same as (3), some weeds and stones	0.035	0.040	0.045	0.050
6. Same as (4), stony sections	0.045	0.050	0.055	0.060
7. Sluggish river reaches, rather weedy or with very deep pools	0.050	0.060	0.070	0.080
8. Very weedy reaches	0.075	0.100	0.125	0.150

[1] Values commonly used in designing.

The values of n were intended to be used only in Kutter's formula but they apply equally well to Manning's formula.

The principal difference between the Kutter and the Manning formulas is the inclusion of S in the former and its omission in

[1] HORTON, ROBERT E., "Some Better Kutter's Formula Coefficients." *Eng. News*, vol. 75.

the latter. An examination of the values of C as given by Kutter's formula, however, shows that for surface slopes greater than 0.0003 the value of C is not affected by the slope. It is these slopes which are generally encountered in practice. Hence, except for flat slopes, the Manning formula may be expected to give identical results with the Kutter formula. Because of its simplicity, the Manning formula would appear to be preferable as, whatever difference there may be in the results obtained by the two formulas, the difference is well within the limits of uncertainty of a proper choice of n in all practical cases.

For ease in computation, tables have been prepared which give values of C for different conditions. These may be found in various hydraulic textbooks and handbooks.[1]

Values of m for use in the Bazin formula are given in the following table:

TABLE XI.—KING'S VALUES[1] OF m TO BE USED WITH BAZIN'S FORMULA

Surface	Best	Good	Fair	Bad
Earth canals in good condition	0.90	1.25	1.60	1.90
Earth canals with weeds, rocks, etc	1.90	2.50	3.15	3.80
Canals excavated in rock	2.50	3.15	3.70	4.20
Natural streams in good condition	1.90	2.50	3.15	3.80
Natural streams with weeds, rocks, etc	3.15	4.40	6.30	8.80

[1] KING, H. W., "Handbook of Hydraulics," McGraw-Hill Book Company, Inc.

It is evident that the proper selection of the roughness coefficient for a natural stream is difficult and uncertain because of the variableness of the lining along the course. Whatever value of n is selected, it is at best a guess or an estimate, if you will, and the discharge determined by the slope method can only be regarded as approximate. For flood discharge, where instrument measurements are impracticable, this method serves excellently to estimate the volume flowing.

In artificial channels, where the lining is known and the cross-section constant, the Chezy formula will give reliable results. It is perhaps in connection with the design of an artificial channel or with the measurement of the discharge of an artificial channel that the Chezy formula should best be associated and reserved for only approximate work in connection with rivers.

[1] KING, H. W., "Handbook of Hydraulics," McGraw-Hill Book Company, Inc.

WEIR MEASUREMENTS[1]

174. Data to Be Collected.—Weir measurements consist in collecting certain data to be used in some weir formula for determining the discharge. These data will include: head on crest of weir; length of crest, if the section is rectangular or trapezoidal; angle of side slopes, if triangular or trapezoidal; nature of crest, whether sharp or broad crested, and if broad crested, the shape of the crest; height of crest above bed of channel; number of end contractions; width of channel of approach; depth of channel of approach; and velocity of approach.

175. Sharp-crested Rectangular Weirs.—Theoretically, the discharge over a sharp-crested rectangular weir will be given by the formula

$$Q = \tfrac{2}{3}\sqrt{2g} \cdot L \cdot H^{3/2}$$

where g = gravity, L = length of crest in feet, and H = head on crest, in feet (Fig. 109).

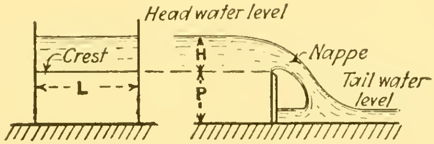


FIG. 109.—Rectangular weir.

This formula has to be modified to allow for the effect of friction at the edges of the section; the top and bottom contractions of the nappe; the side contractions, if the crest length is less than the width of the channel of approach; and the effect of the velocity of approach. The determination of constants and form of the formula to allow for these factors has been the object of several investigators. Their work, together with their published formulas, are discussed in the various textbooks on hydraulics. In this chapter, only the formulas will be given with such information as is necessary to explain the conditions under which the formulas may be relied on to give best results.

176. Empirical Formulas.—Many empirical formulas have been published for determining the discharge over rectangular weirs and, in general, they have been based on the experiments of Francis (1852), Fteley and Stearns (1877–1879), and Bazin (1886).

[1] See Chap. IV for description of Weir station.

The formulas proposed by these men are the following:

Francis: $$Q = 3.33L[(H + h)^{3/2} - h^{3/2}],$$

in which, 3.33 = a constant which includes the effect of friction and the top and bottom contractions of the nappe.

L = Length of crest, in feet.

H = Head on crest, in feet.

$h = \frac{v^2}{2g}$, known as the velocity head, due to the velocity of approach, v.

This formula applies when there are no end contractions, or in other words, for a suppressed weir.

When there are end contractions, Francis determined from his experiments that the crest length, L, was reduced by an amount of $\frac{H}{10}$ for each contraction so that the modified length of the crest would be

$$L' = L - \frac{nH}{10}$$

n, being the number of end contractions.

Fteley and Stearns: $Q = 3.31\,L(H + fH)^{3/2} + 0.007L$, in which L, H, and h have the same meaning as in the Francis formula. f is a multiplier and equals 1.5 for suppressed weirs, and 2.05 for contracted weirs. The term, $0.007L$, was added to make their formula give consistent results with the Francis formula for low heads.

In the above form, the discharge is for a suppressed weir. The effect of end contractions is allowed for in the same way as in the Francis formula.

Bazin: $$Q = \left[0.405 + \frac{0.00984}{H}\right]\left[1 + 0.55\left(\frac{LH}{A}\right)^2\right]L\sqrt{2g}H^{3/2},$$

in which L and H have the same meaning as before. A is the area of the cross-section of the channel of approach. This term involving $\left(\frac{LH}{A}\right)$ is inserted in the formula to allow for the effect of the velocity of approach. It is evident that this simplifies the process of measuring the discharge, for A is more easily obtained than v.

Bazin's experiments were made with a suppressed weir so that his formula was derived expressly for that type of weir.

If the formula is to be used for a contracted weir, the Francis method of reduction may be used.

King: Prof. H. W. King investigated the flow of water over sharp-crested weirs, using as a basis the data obtained by Francis, Fteley and Stearns, and Bazin, and derived the following formula for suppressed weirs, which has much merit:

$$Q = 3.34LH^{1.47}\left[1 + 0.56\left(\frac{LH}{A}\right)^2\right],$$

in which L, H, and A have the same meanings as before. L should be reduced by $\frac{H}{10}$ for each end contraction, when used for a contracted weir.

From his study,[1] King found that the results obtained by the use of his formula agreed more closely with those obtained by Bazin than did those of either Francis or Fteley and Stearns.

177. Choice of Formula.—These formulas can be relied on to give their best results when the conditions of measurement are similar to those under which the experiments were made. In order that an idea may be had of the range of conditions under which the experiments were carried on, the following table is given:

TABLE XII.—SHOWING VARIOUS CONDITIONS UNDER WHICH WEIR EXPERIMENTS OF FRANCIS, FTELEY AND STEARNS, AND BAZIN WERE MADE

Name	Head, feet	Height of crest, feet	Length of crest, feet	Width of channel, feet	Velocity of approach, feet per second
Francis	0.63–1.6	2.0 –4.6	8.0–10.0	10 –14	0.22 –1.0
Fteley-Stearns	0.07–1.6	0.5 –6.6	2.3–19.0	5– 19	0.023–2.4
Bazin	0.19–1.9	0.79–3.7	1.6– 6.6	1.6– 6.6	0.07 –1.6

The formula which is used should be limited to the range of experimental data on which it is based. Where the velocity of approach is less than 1 ft. per second, Francis' formula is suitable for heads from 0.5 ft. to 2 ft. and Fteley-Stearns' formula for heads from 0.07 to 0.5 ft. For higher heads, Bazin's or King's formula give good results. Errors of 1 per cent, for favorable

[1] KING, H. W., "Handbook of Hydraulics," McGraw-Hill Book Company, Inc.

conditions, and 5 per cent for unfavorable conditions, may be expected.

178. Requirements for Maximum Accuracy.—The following points should be observed in making weir measurements if the maximum accuracy is to be obtained.

Head.—In measuring the heads on a weir small quantities are being dealt with so that the measurements should be done as accurately as possible. For this purpose, a hook gage should be used and should be located in a stilling box connected by pipes with the channel to cut off any wave action. With this gage, the head may be read to 0.001 ft. The relation between the zero of the scale and the elevation of the crest may be found by

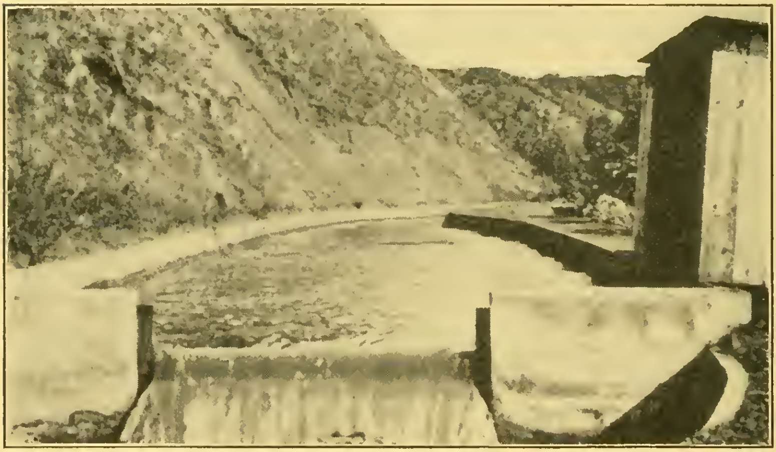


FIG. 110.—Weir gaging station, Wakiawa Reservoir, Ditch Oahu, Hawaii. (*From U. S. Water Supply Paper* 318).

careful leveling. The head should be taken as the mean of at least twenty readings made at equal intervals of one-half minute or so, in order to allow for any fluctuations due to wave action.

The minimum head to be measured should not be less than 0.2 ft. If the head is less than this amount, there is a tendency for the nappe to cling to the downstream face of the weir. Because of the experimental limits used in the derivation of the formulas, the head should not be greater than 1.5 ft. The size of the head on a contracted weir can be varied by varying the length of the crest so as to obtain a suitable head for measuring.

The head should be measured far enough upstream from the weir to be free from the effect of the surface drop as the weir

is approached. This distance varies from $2\frac{1}{2}H$ to $3H$. However, in order that the formula may be more truly applicable, the distance upstream should be the same as that used in the experiment on which the formula is based. For Francis, the

FIG. 111.—Quakish Lake Dam of Great Northern Paper Co., near Millinocket, Maine. Dam is used as a weir to measure the discharge from the lake. The coefficient in the formula $Q = CLH^{3/2}$ is variable, its value having been obtained from experiments for different values of H. (*From U. S. Water Supply Paper* 279.)

distance was 6 ft.; for Fteley and Stearns, it was 16.4 ft.; and for Bazin, it was 16 ft.

Crest.—The crest should be level and sharp edged. Its length should be greater than $3H$. If there are end contractions, they

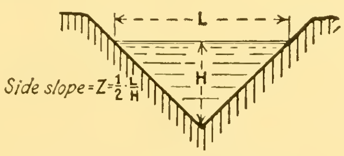


FIG. 112.—Triangular weir.

should be complete. This will be secured if there is a clearance of at least $3H$ between the end of the crest and the side of the channel.

Nappe.—The nappe should be completely aerated; *i.e.*, air should have free access to the space beneath the nappe. A

vacuum in this space will cause an increase in discharge as shown by Bazin's experiments. For the contracted weir there is no need for any special provision for admitting air as there is ample opportunity for air to get in under the nappe.

Face of Weir.—The upstream face of the weir should be in a vertical plane as an upstream inclination will reduce the discharge and *vice versa* a downstream inclination will increase the discharge.

179. Triangular Weirs.—The theoretical formula for the discharge over a triangular weir is

$$Q = \tfrac{4}{15}\, L\sqrt{2g}H^{3/2},$$

in which L is the width of the water surface (Fig. 112).

Professor Thompson[1] deduced experimentally a value of $C = 0.593$ for heads from 0.15 to 0.60 ft. Using this coefficient, the actual discharge would be

$$Q = 1.27LH^{3/2},$$

or, if written in terms of the slope of the sides, designated by z

$$Q = 2.54zH^{5/2}.$$

For a right angled notch $z = 1$ and

$$Q = 2.54H^{5/2}.$$

King[2] suggests instead the formula $Q = 2.52H^{2.47}$ as being more accurate.

180. Trapezoidal Weirs.—The discharge over a trapezoidal-shaped weir may be considered as equal to the combined dis-

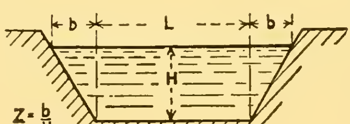


Fig. 113.—Trapezoidal weir.

charge of a suppressed rectangular weir of length L and a V-notch weir with side slopes $\frac{b}{H} = z$ (Fig. 113). The discharge, under this assumption, woud be

$$\begin{aligned} Q &= 3.33LH^{3/2} + 1.27 \times 2 \times bH^{3/2} \\ &= 3.33LH^{3/2} + 2.54bH^{3/2} \\ &= (3.33L + 2.54b)H^{3/2}, \end{aligned}$$

[1] *British Association Reports*, 1861.

[2] King, H. W., "Handbook of Hydraulics," McGraw-Hill Book Company, Inc.

which combined Francis' suppressed weir formula and Thompson's triangular formula. Experience has shown that this formula will give discharges which are too large.

Cippoletti, an Italian engineer, concluded from some studies of Francis' experiments that a ratio of $\frac{b}{H} = 0.25$ would approximately offset the effect of end contractions of a rectangular weir and that a coefficient of 3.367 would be better than 3.33 of Francis. Hence, the formula for the Cippoletti weir, $\left(\frac{b}{H} = \frac{1}{4}\right)$

$$Q = 3.367LH^{3/2}.$$

181. Broad-crested Weir.—Where the weir is not of the sharp-crested type but is broad crested as, for example, the spillway of a dam, the weir formulas given above may be used, provided appropriate values of the coefficient are used. The formula is $Q = CLH^{3/2}$.

Robert E. Horton has given in *U. S. Water Supply Paper* 200 the values of C applicable to broad-crested weirs of various cross-sectional shapes between certain heads. These values were obtained from an exhaustive study of two series of experiments on broad-crested weir coefficients made independently by Bazin, in 1897, and Rafter, in 1898. This Paper should be referred to when making a choice of a coefficient. Various types of cross-sections are given and the one which simulates most closely the section of the weir to be used in measuring should be selected to determine the appropriate coefficient.

If the particular shape of crest is not included in Horton's table, it may be necessary to build a model of the crest and determine the coefficient experimentally.

For approximate values of discharge, C may be taken as 2.64 for sharp-edged broad-crested weirs, 2 ft. or more wide, having heads sufficient to cause the nappe to fall free from the downstream face, but not so great as to cause the nappe to leave the crest; and as 3.8 for broad-crested weirs with rounded upstream edges. The latter type corresponds to the ordinary spillway section.

CHAPTER XIII

CONSTRUCTION AND USE OF STATION RATING CURVE

182. Analysis of Discharge Measurements.—The station rating curve will be defined by measured discharges plotted against corresponding gage heights. The closeness with which these plotted discharges will define the discharge curve will depend largely on the accuracy of the individual discharge measurements and their distribution in range. If the control is permanent and the discharge measurements are accurate, the points should plot on a smooth curve; but due to inaccuracies of one sort or another, entering into the measurements, the points may not plot on a smooth curve and, consequently, the curve will be drawn among the points so as to average them.

The variations from the average curve of measurements, made under reasonably satisfactory conditions, should be small and, as likely as not, the plus variations will average the minus variations. On the other hand, the discharges may not define a smooth curve because some of them are inaccurate. The inaccuracy may be in the observations, in the computations or in the plotting. Therefore, in order to determine the accuracy of the plotted points, each individual measurement should be studied. This study is best made by analyzing the measured discharge into its two component parts, *viz.*, the area and the mean velocity. After separating the discharge into these two factors, each factor can be studied to determine its individual accuracy. It will be much easier to study the relation of these separate factors to the stage than it will be to study the relation of their product to the stage.

A study of the relation of each of these factors to the stage is most readily accomplished by means of curves showing, respectively, the relation between areas and gage heights, and the relation between mean velocities and gage heights. The shape of these curves will be affected by the area of the channel cross-section, the surface slope, and the channel lining. Only the first factor will affect the shape of the area curve but all three will affect the shape of the velocity curve.

183. Area Curve.—In preparing an area curve it is desirable, first, to determine the profile of the channel section. This profile should be constructed from carefully made soundings obtained at low water and, in addition, a seriės of levels extending from the low water level up the banks to the point of high water.

With the aid of this profile the area corresponding to any gage height can be determined. The number of areas necessary in the construction of an area curve will depend on the shape of the profile. In general, the areas determined at gage heights where there are breaks in the profile will afford sufficient accuracy. However, it may be advisable to determine the areas at intermediate gage heights to insure greater accuracy. After these areas have been determined, they can then be plotted against their corresponding gage heights and a smooth curve drawn through them.

Since the area is a function of the width and depth of the water, the shape of the curve will depend on the variation in width with the change in depth. This variation may be such that the width will either (*a*) increase with the depth, (*b*) remain constant regardless of the depth, or (*c*) decrease with the depth, the last being the case of over hanging banks. These three cases will cause, respectively, either a curve concave to the axis of areas, one that is straight, or one that is convex to the axis of areas. The first case is the most common one for natural streams.

In developing an equation for the area curve it will be necessary to make an assumption as to the shape of the cross-section. The profile of the bed and sides may be assumed to be made up of either a series of short straight lines or a series of parabolic arcs with vertical axes, being short or long, of different parameters, and convex or concave to the water surface. An equation based on the first assumption will be much less complicated than one based on the second and if the lengths of the lines are taken very short, the area obtained by means of such an equation will be accurate enough for all practical purposes.

F. W. Hanna, in *U. S. Water Supply Paper* 146, has analyzed very fully the mathematical features of the area curve, based on the rectilinear assumption. He considered three possible types of cross-section of a stream where shapes are shown in Fig. 114.

If A = area of the cross-section,

B = base width,

D = depth of water,

C = area below the base of any trapezoid,

θ and φ the complements of the slopes of the sides of a trapezoid,

then, in Fig. 114*a*, $A = BD$. (1)

In Fig. 114*b*, $A = BD + \frac{1}{2}D^2 (\tan \theta + \tan \varphi)$. (2)

In Fig. 114*c*, $A = BD + \frac{1}{2}D^2 (\tan \theta + \tan \varphi) + C$. (3)

Equation (1) would be that of a straight line and would apply to a rectangular section.

Equation (2) would be that of a parabola and would apply to a trapezoidal section.

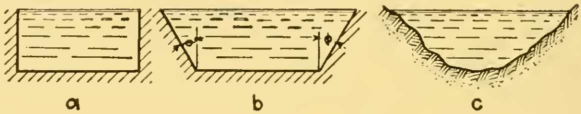


FIG. 114.—Typical channel cross-sections.

Equation (3) would be that of a series of parabolic arcs with varying parameters and would apply to a section with an irregular profile.

By differentiating Eqs. (1), (2), and (3), there results, respectively,

$$\frac{dA}{dD} = B. \tag{4}$$

$$\frac{dA}{dD} = B + (\tan \theta + \tan \varphi)D. \tag{5}$$

$$\frac{dA}{dD} = B + (\tan \theta + \tan \varphi)D. \tag{6}$$

The ratio $\frac{dA}{dD}$ will be the slope of the area curve. For the rectangular section, the slope is constant. For the trapezoidal section, since θ and φ are constant, the slope will become flatter with an increase in depth. It will be noticed that the slope, when D is zero, will be equal to the base width. For the irregular section, the slope will depend on the slope of the banks as well as the depth.

By differentiating Eqs. (4), (5), and (6) and multiplying by A, there results

$$\frac{d^2A}{dD^2} \times A = 0. \tag{7}$$

$$\frac{d^2A}{dD^2} \times A = 2(\tan \theta + \tan \varphi)A. \tag{8}$$

$$\frac{d^2A}{dD^2} \times A = 2(\tan \theta + \tan \varphi)A. \tag{9}$$

From the theory of curvature, whenever the value of $\frac{d^2A}{dD^2} \times A$ is positive, the curve is concave towards the A axis. Consequently, as the expression is positive in each of the above cases, the area curve is concave to the A axis in each case.

From the above discussion, the following points are to be noticed:

1. The profile of the channel section should be carefully examined before constructing an area curve.
2. For a rectangular section the area curve will be a straight line, having a slope equal to the width of the section.
3. For a trapezoidal section the area curve will be a parabola and may be constructed by means of several controlling points.
4. For an irregular section—the ordinary river channel section—the curve is best drawn so as to average the points plotted for numerous gage heights.

184. Slope of Area Curve at Any Point.—The slope of the area curve at any point may be obtained in the following manner.

If, in Fig. 115, we let

W = width of the stream at any stage,
A = corresponding area of the stream,
D = corresponding water depth above the lowest point of the bed,

then, for any small change, dD, in the depth, D, the area will be changed by an amount, $dA = W \times dD$, inasmuch as the width, W, at the depth, D, may be taken as constant for the differential rise, dD.

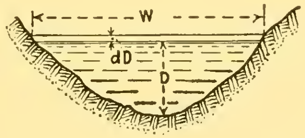


Fig. 115.

Rewriting, $$W = \frac{dA}{dD},$$

which is the slope of the area curve. Hence, for any stage, the width of the stream is equal to the slope of the area curve at that stage.

Practical use may be made of this fact in the construction of the area curve. Since the actual slope of the area curve, on the plot, will depend upon the scales used, it is necessary to adopt a method for laying off the tangent so that the scales may be taken into account. If we consider $dD = 1$ ft., then $W = dA$. Consequently, by laying off 1 ft. from the plotted point along the gage axis to the scale of gage heights and at the end of this

distance, laying off W along the area axis to the scale of areas, the line joining the end of the width and the plotted point will have the desired slope.

It should be noted that, for all sections except those with flat bottoms, the curve is tangent to the gage axis at the origin; but that for flat bottoms the curve is not tangent to the gage axis at the origin but makes an angle with the gage axis whose slope is equal to the bottom width.

Determining in this manner the slopes at the several plotted points, the curve may be drawn in so that it is kept parallel to these tangents.

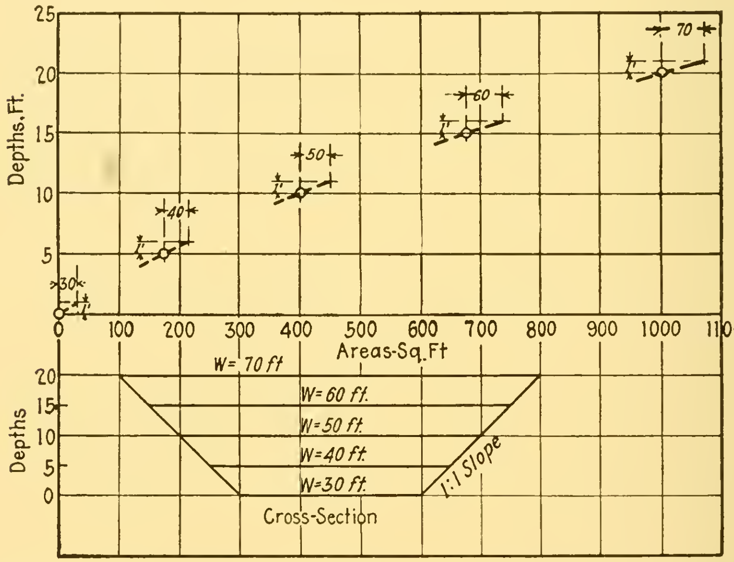


FIG. 116.—Illustrating method of constructing area curve with the aid of tangents drawn in at various gage heights.

This method is illustrated in Fig. 116 where a trapezoidal section has been chosen and the slope of the area curve determined at every 5-ft. depth. The curve, in order to make the plotting of the tangents clearer, has not been drawn in.

185. Further Analysis of the Area Curve.—The area curve may be further analysed by considering Fig. 117, in which TT' is drawn tangent to an area curve at a point where the depth is D and the area, A.

The equation for the curve may be expressed in the form,

$$A = K_1 D^x,$$

where K is a constant and x an exponent, depending for its value on the slope of the banks.

$x = 1$ for vertical banks.
$1 > x < 2$ for banks concave upward.
$x = 2$ for straight sloping banks.
$x > 2$ for banks convex upward.

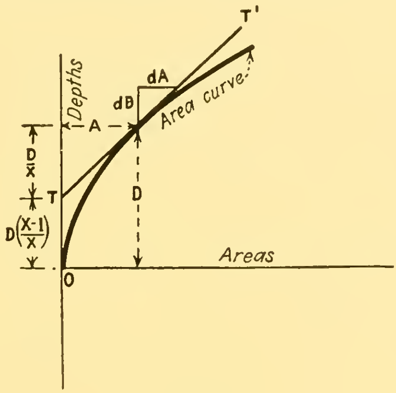


FIG. 117.

Then, from Fig. 117, we may write the proportion,

$$\frac{A}{NT} = \frac{dA}{dD}. \tag{10}$$

Substituting, $$A = K_1 D^x,$$

and $$\frac{dA}{dD} = K_1 \cdot x \cdot D^{x-1}, \tag{11}$$

from which $$NT = \frac{D}{x}. \tag{12}$$

Therefore, $$\overline{OT} = D\left(\frac{x-1}{x}\right), \tag{13}$$

which is the distance above the origin that the tangent will intersect the axis of the depth.

Measuring OT on the plot and knowing D, x may be calculated and K_1 determined, thus obtaining the equation of the curve. It should be observed, however, that this is only theoretical, as x and K_1 will vary for different depths, but such an equation may serve as a basis for a study of the curve.

186. Mean-velocity Curve.—The mean-velocity curve will show the relation of the mean velocity to the stage. Points on the curve are located from measurements made at different stages covering the range of gage heights possible at the section. The velocity for any stage will be determined from the ratio of Q to A for that stage.

Generally, the points will plot so that a curve is readily defined but, in some instances, the points will be quite discordant and a knowledge of the shape and properties of the velocity curve is necessary in order to draw the curve correctly. Using the formula, $V = C\sqrt{RS}$, as a basis for such a curve, we may rewrite it thus, $V^2 = C^2RS$. Calling A the wetted area and P the wetted perimeter, $V^2 = C^2\frac{A}{P}S$. Substituting[1] for A, the values corresponding to the several shapes of cross-section shown in Fig. 114a, b, and c, there results,

$$V^2 = C^2S\frac{BD}{P}. \tag{14}$$

$$V^2 = C^2S\frac{\left[BD + \frac{D^2}{2}(\tan\theta + \tan\varphi).\right]}{P}. \tag{15}$$

$$V^2 = C^2S\frac{\left[BD + \frac{D^2}{2}(\tan\theta + \tan\varphi) + C\right]}{P}. \tag{16}$$

C, S, and P may be assumed to be constant, an assumption which will be true if there is no change in roughness, a uniform cross-section, and a permanent flow.

Equation (14) is that of a parabola, Eq. (15) is that of an hyperbola with a horizontal axis, and Eq. (16) is that of a series of hyperbolas with varying eccentricities and usually with horizontal axes.

Considering the hydraulic radius to be equal to the area divided by the width, which is permissible when the stream is quite broad compared to the depth, the Chezy formula becomes

$$V = CS^{1/2}d^{1/2}, \tag{17}$$

where d is the mean depth.

Since d is a function of D,

$$V = CS^{1/2}D^{1/2}, \tag{18}$$

and assuming C and S constant, we may further write

$$V = K_2D^{1/2}. \tag{19}$$

[1] See Hanna's analysis in *U. S. Water Supply Paper* 146.

The graph of this curve will be as shown in Fig. 118. Differentiating,

$$\frac{dV}{dD} = \frac{K_2}{2D^{1/2}}, \tag{20}$$

which is the slope of the curve at any point. The tangent will meet the axis of velocities at a distance from the origin equal to $\frac{V}{2}$ and the axis of the gage heights at a distance $-D$.

From these assumptions, the curve should be tangent to the axis of velocities at the origin. In the case, however, where there is zero flow with ponded water at the gaging section, the curve will reverse at low stages and approach the origin convex to the gage axis. This change in slope is controlled by the amount of ponded water at the gage, the roughness of the bed, and the form of the control.

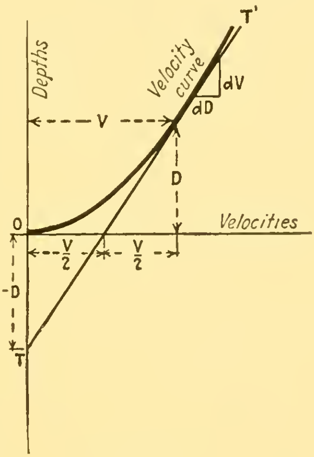


FIG. 118.

187. Discharge Curve.—The discharge curve may be constructed in one of three ways, *viz.*, (*a*) by plotting the measurements of discharge against gage heights, (*b*) discharge against $A\sqrt{d}$, or (*c*) on logarithmic paper, with discharges and gage heights as coordinates. The three methods will now be considered.

1. *Discharge vs. Gage Height.*—This method is the most common one and consists in plotting the several measured discharges against their corresponding gage heights. These points will define the position and shape of the discharge curve. If the measurements have been accurately made, the curve drawn among the points should be smooth (Fig. 119).

The area curve and the mean velocity curve should be drawn in conjunction with the discharge curve. These can be used in testing the accuracy of the discharge measurements. Inasmuch as the mean velocity is determined by dividing the discharge by the area, the mean velocity curve can be only approximate. With the area curve accurately drawn, however, it is possible to determine the error in the discharge. These two curves will

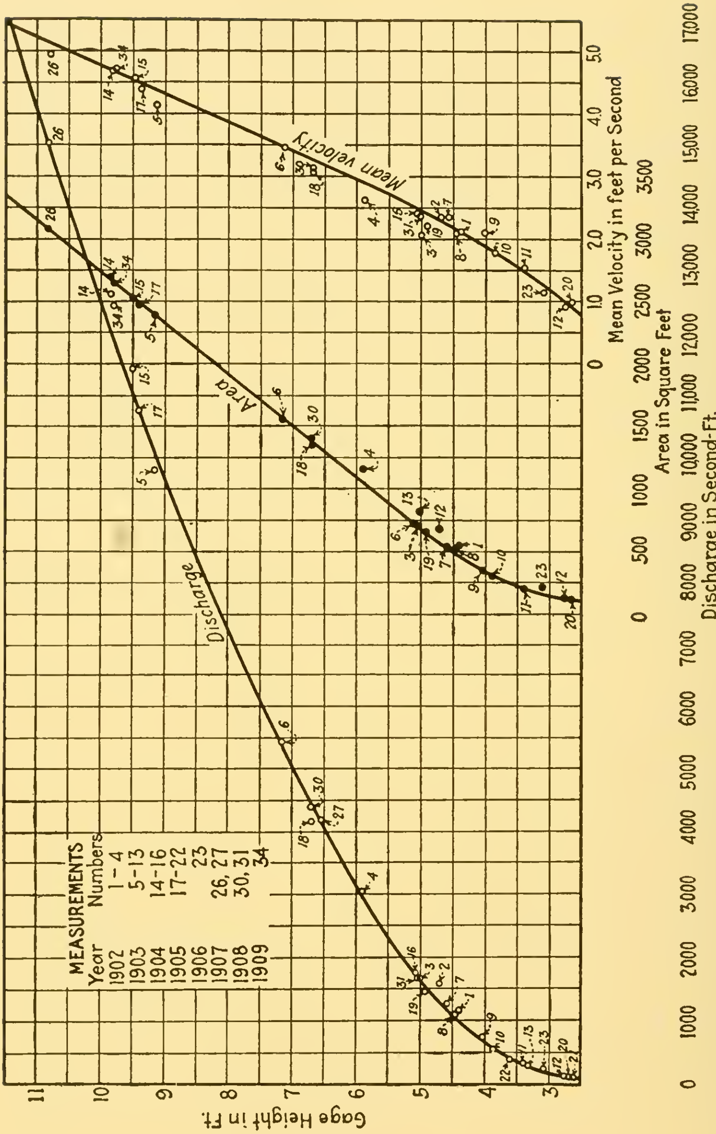


FIG. 119.—Discharge, area, and mean velocity curves of Mattawamkeag River at Mattawamkeag, Maine. (*From U. S. Water Supply Paper* 279.)

also be useful in obtaining any point on the discharge curve where no actual discharge measurement has been made.

If the several individual discharges are investigated in this way before plotting, erroneous measurements may be corrected and the relative accuracy of each measurement be determined so that the curve may be correctly drawn.

2. *Discharge vs.* $A\sqrt{d}$.—J. C. Stevens[1] found that a discharge curve might be drawn as a straight line if $A\sqrt{d}$ was used as one of the coordinates. Since it is safer to extend a straight line than a curved one, a rating curve showing the relation between the discharge, Q, and $A\sqrt{d}$ will be suitable in cases where it is necessary to extend the curve beyond the observed points. In this method ordinary cross-section paper is used.

The linear relation between Q and $A\sqrt{d}$ is shown by considering the Chezy formula for discharge,

$$Q = AC\sqrt{RS}.$$

This may be written

$$Q = C\sqrt{S} \cdot A\sqrt{R}.$$

The product of C and $\sqrt{S}$ is practically constant for the higher stages, when R is greater than 3.28, so that for high stages and deep streams, it will be approximately true that

$$Q \propto A\sqrt{R}.$$

Since $\frac{A}{P}$ is very nearly equal to $\frac{A}{W}$, the latter ratio, which is the mean depth, d, may be put in place of R and no appreciable error will result in the plotting. This substitution for d and R facilitates the work because d is more easily obtained than R.

For convenience in using the curve, an auxiliary curve is drawn with gage heights and $A\sqrt{d}$ as coordinates. By plotting this auxiliary curve so that the same axis of $A\sqrt{d}$ is used for both curves, the observed gage height may be applied to the curve of $A\sqrt{d}$ *vs.* gage heights and by running across on the intersected value of $A\sqrt{d}$, to the curve of Q *vs.* $A\sqrt{d}$, the corresponding value of Q may be read on the scale of discharge.

The curves are best constructed in the following manner:

1. Plot the cross-section of the stream.

2. From the cross-section, prepare a table of widths, areas, mean depths, and values of $A\sqrt{d}$ for each foot or half-foot of gage heights as abscissae.

[1] *Eng. News.*, vol. 58.

3. Plot measured discharges against corresponding values of $A\sqrt{d}$, using Q as abscissae and $A\sqrt{d}$ as ordinates, and pass a straight line through the plotted points.

4. Plot the values of computed $A\sqrt{d}$ against corresponding gage heights, having the gage heights laid off as abscissas and the value of $A\sqrt{d}$ as ordinates, using the same scale as in plotting the curve of Q *vs.* $A\sqrt{d}$. A smooth curve is then drawn through these points.

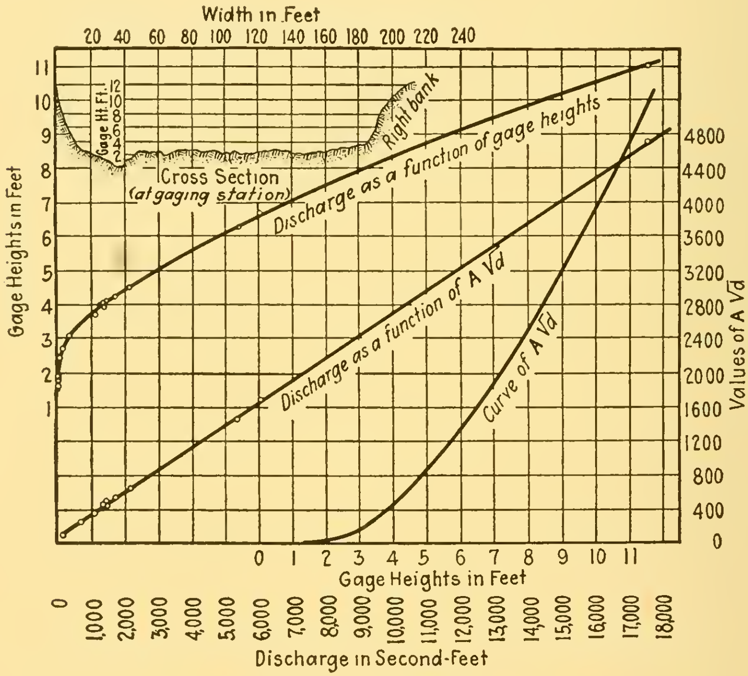


FIG. 120.—Discharge curves for Umatilla River at Umatilla, Oregon.

For the case of ponded water, at the gage height corresponding to zero discharge the corresponding value of $A\sqrt{d}$ should be subtracted from the tubular values of this quantity before plotting.

Figure 120 illustrates the manner of arranging the curves for most convenient use. In this figure the curve of Q *vs.* gage height is also given. For any gage height the same discharge should be obtained regardless of which curve is used. For example, the

discharge is 4600 cu. ft. per second at a gage height of 5.8 ft., whether obtained with the aid of the curve of $A\sqrt{d}$ *vs.* gage heights or directly from the curve of Q *vs.* gage heights.

3. *Logarithmic Plot.*—Logarithmic paper may be used and the discharge plotted against gage heights. The points will plot on a straight line and therefore this method is suitable where it is necessary to extend the curve above the measured discharges.

188. Rating Table.—After the rating curve has been drawn it is desirable to scale off values for each tenth, half-tenth, or hundredth of gage height and arrange them in tabular form. The closeness with which the gage is to be read, together with the probable

Gage height, feet	Discharge, sec.-ft.	Gage height, feet	Discharge, sec.-ft.	Gage height, feet	Discharge, sec.-ft.	Gage height, feet	Discharge, sec.-ft.
2.50	86	3.90	590	5.30	2,080	7.40	5,920
2.60	100	4.00	660	5.40	2,220	7.60	6,360
2.70	114	4.10	736	5.50	2,360	7.80	6,810
2.80	134	4.20	818	5.60	2,500	8.00	7,270
2.90	160	4.30	906	5.70	2,660	8.20	7,750
3.00	190	4.40	1,000	5.80	2,820	8.40	8,230
3.10	223	4.50	1,100	5.90	2,980	8.60	8,730
3.20	258	4.60	1,210	6.00	3,160	8.80	9,230
3.30	295	4.70	1,320	6.20	3,520	9.00	9,750
3.40	334	4.80	1,440	6.40	3,900	10.00	12,520
3.50	375	4.90	1,560	6.60	4,280	11.00	15,570
3.60	420	5.00	1,690	6.80	4,680	12.00	18,620
3.70	470	5.10	1,820	7.00	5,080	13.00	21,670
3.80	525	5.20	1,950	7.20	5,490	14.00	24,720

FIG. 121.—Rating table for Mattawamkeag River at Mattawamkeag, Me., 1902–1909.

accuracy of the rating curve, will determine at what intervals of gage heights the discharges should be read from the curve.

The scaled values may be corrected so as to be consistent among themselves by taking first differences, and, sometimes, second differences and adjusting them so that they will either be constant or will gradually increase with increasing gage heights.

Figure 121 shows a rating table constructed for the rating station at Mattawamkeag, Me., on the Mattawamkeag River. The area curve, mean velocity curve, and discharge curve are shown in Fig. 119.

189. Accuracy of Daily Stream-flow Records.—It will be remembered that the object of constructing a station-rating curve is to afford a simple means for determining the average daily flow of a stream. The daily discharge is to be obtained by applying the daily gage height to the rating curve. The accuracy of the discharge thus determined will be governed by (*a*) the accuracy of the curve and (*b*) the accuracy of the daily gage height which is applied to the curve.

The accuracy of the rating curve has been discussed in earlier articles. It will depend on (*a*) the permanence of the relation of the stage to the discharge which is governed, in turn, by the permanence of the control, and (*b*) the accuracy of the shape and position of the rating curve which is governed by the accuracy of the individual discharge measurements.

The accuracy of the daily gage height will depend on (*a*) the closeness of the reading, (*b*) the frequency of the reading, and (*c*) the method of applying the daily gage height to the rating curve.

(*a*) The closeness of the reading, necessary for accuracy, will depend on the stage, the closer readings being required at the lower stages.

The gage is generally read either to tenths, half-tenths, quarter-tenths, or hundredths. The closer the gage is read, the less will be the error in the corresponding discharge. The U. S. Geological Survey has chosen, as a limiting figure, 2 per cent for the allowable average error for staff and drain gages and 1 per cent for recording gages in the determination of daily discharge due to lack of refinement. The maximum error in daily discharge will be twice as great, or 4 per cent, and the resulting error in the determination of the mean discharge for the month will be approximately 0.33 per cent.

The required closeness of the gage reading at any stage can be determined by finding from the rating table the difference in gage reading at that stage which will cause the percentage difference in corresponding discharges not to exceed the limiting percentage.

A practical method, recommended in *U. S. Water Supply Paper* 400-D, for determining the limits of use of the gage heights, is as follows:

Find the stage at which the difference in discharge per tenth is 8 per cent of the discharge at that stage. Gage heights above this stage should be used to tenths.

Find the stage at which the difference in discharge per tenth is 16 per cent of the discharge at that stage. Gage heights below this stage should be used to hundredths.

Gage heights between the two stages thus found should be used to half-tenths.

This requirement for close reading of the gage presumes a sensitive station where a change of 1 per cent in the discharge produces a change in the gage height which can be read on the gage. Of course, if the stage-discharge relation is not a sensitive one, then close reading of the gage is of no purpose.

(*b*) The necessary frequency of gage readings will depend on the rate of fluctuation of the stage of the stream. This must be determined by comparing the daily discharge obtained by using one daily reading or the mean of two daily readings, with the daily discharge obtained by hourly readings.

(*c*) The method of applying the daily gage height to the rating curve will depend on the degree of curvature of the rating curve. If the mean daily gage height is used, the discharge obtained will be too small owing to the fact that a curvilinear relation exists

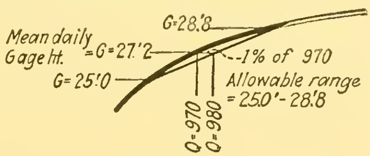


Fig. 122.

between the gage height and the discharge. The amount of error in the discharge, read from the curve at the mean daily gage height, will be governed by the sharpness of the curve and the daily range in gage heights. If the error is greater than that allowable, it will be necessary to use smaller time intervals and average the discharges obtained by applying the means of these shorter intervals to the rating curve.

In *U. S. Water Supply Paper* 400-D, it is recommended that the allowable daily range in stage be such as to limit the maximum error to 1 per cent. This allowable range can be determined graphically by intersecting the curve with a chord in such a way that the horizontal distance, measured to the scale of discharge, from the midpoint of the chord to the curve shall not be greater than 1 per cent of the discharge at the gage height corresponding to the middle point. The gage heights at the ends of the chord

will be the upper and lower limits of the daily gage heights (Fig. 122).

A table of allowable ranges in daily gage heights corresponding to a mean daily gage height can be constructed in order to facilitate the determination of the range in gage heights.

The U. S. Geological Survey uses a discharge integrator for determining the mean daily discharge of streams where there is considerable fluctuation in stage. The integrator is provided with a flexible steel ribbon which can be bent to conform to the shape of the rating curve and held in that position. By moving the index pointer along the graph of gage heights in the same manner as is done in the use of a planimeter, the discharge is automatically integrated and is registered on dials.

190. Use of Discharge Curves for Streams of Unstable Bed.—In the cases of streams where the beds and banks are continually shifting due to scour or silting, the relation between the discharge and the gage height is not constant. In such cases, the discharge curve must be used with special care and adjustments made to allow for the changing relation. This is done by making discharge measurements every few days and determining the discharge on intervening days by interpolation. Two methods for making these interpolations have been devised and are known as the Stout and Bolster methods from the names of the men who devised them.

191. Stout Method.—The Stout method requires a station rating curve constructed from measurements made at various stages. This curve can be regarded as only approximate owing to the unstable nature of the channel. The measurements used in constructing the curve should be made, preferably, in a chronologically consecutive order and not too widely separated in time. This is best done during a period of rising or falling stage.

Succeeding measured discharges, when plotted, will fall on either side of the rating curve, depending on the nature of the shift in the bed of the channel. The differences between gage heights as actually observed and as found from the rating curve to correspond with the measured discharge, are plotted as ordinates and the time intervals as abscissae. Through these plotted points a smooth curve is drawn and gage corrections for intervening days are read from the curve. Applying these corrections to the observed gage heights corresponding to these

intervening days, the discharge may be read from the rating curve.

For example, let Curve *A* in Fig. 123 be a standard rating curve constructed from measurements made on Apr. 1, 3, 8, and

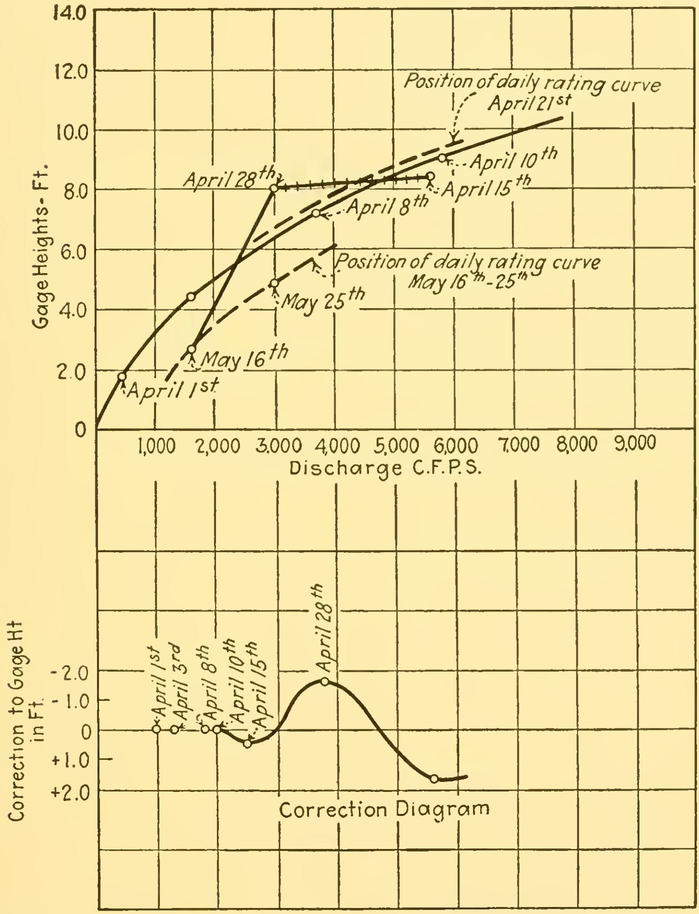


FIG. 123.—Illustrating the use of the Stout and Bolster methods for correcting gage readings at gaging stations where the beds are unstable.

10. On Apr. 15, the discharge was measured and found to be 5600 cu. ft. per second. The corresponding gage height was 8.4 ft. Similarly, on Apr. 28, Q = 3000 cu. ft. per second, gage = 8.0; on May 16, Q = 1650 cu. ft. per second, gage = 2.7 ft. To get these discharges from the rating curve the corresponding

gage heights must be corrected, respectively, by +0.5 ft., −1.6 ft., and +1.7 ft.

Plotting this series of corrections as ordinates against days as abscissae, the correction diagram is drawn. This diagram will give the probable correction to apply to the gage height on any intermediate day.

For example, suppose the gage reading on Apr. 21 was 8.0 ft. The correction would be −0.3 ft. and the corrected gage height would be 7.7 ft. Applying this gage height to the rating curve, the discharge is found to be 4200 cu. ft. per second.

192. Bolster Method.—In the Bolster method one or more standard rating curves are drawn through points plotted in chronologically consecutive order. The measurements may extend over the entire year and there may be several groups of points which will define their own rating curves. These curves will be standard for all gage readings made between the first and last days whose gage heights were used in constructing the curve. For days intervening, the position of the rating curve may be found by joining the points representing consecutive measurements by a line and dividing the line into as many equal intervals as there are days intervening between the days of measurement. The rating curve may then be raised or lowered parallel to itself until it passes through the point of division on the line. The discharge will then be read from the rating curve while in this position by applying the observed gage reading.

For example, suppose the gage reading on Apr. 21 is 8.0 ft. No measurements have been made between Apr. 15 and Apr. 28. Drawing a line between the points of Apr. 15 and 28 and dividing the line into thirteen equal parts, the rating curve is moved upward parallel to itself until it passes through the point representing Apr. 21. In this position, a gage reading of 8.0 gives a discharge of 4180 cu. ft. per second, which is the assumed discharge on Apr. 21.

Instead of drawing the rating curve through each point, the curve together with a vertical reference line can be drawn on a piece of tracing cloth and this curve moved along a corresponding vertical reference line on the paper until the curve passes through the point. The discharge can then be read from the curve.

193. Effect of Changing Stage.—When a river is rising at a fast rate, the velocity, and consequently the discharge, is greater than at the same stage when the flow is constant. Also, when

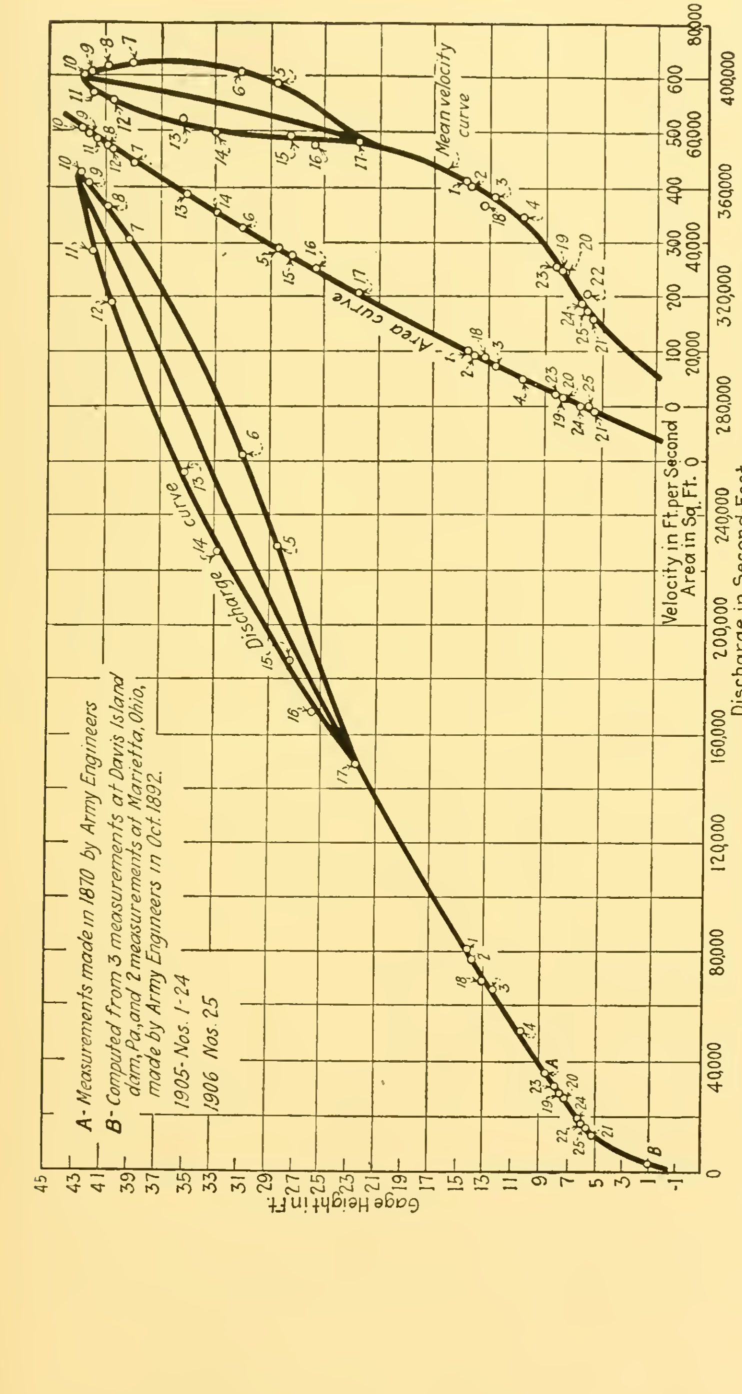


FIG. 124.—Discharge, area, and mean velocity curves for Ohio River at Wheeling, W. Va. (*From U. S. Water Supply Paper* 345.)

falling at a fast rate, the velocity and discharge are smaller (see Fig. 124). Consequently, the relation between discharge and gage height for constant stage will not hold for the case of changing stage and it is necessary to modify the method used in estimating discharges. At any gage height the area and hydraulic radius are constant, regardless of the velocity, providing no scour or silting has occurred. Hence, the increase in velocity must be due to an increase in slope. This follows from the equation of velocity, $V = C\sqrt{RS}$. The coefficient C will vary with the nature of the lining, the hydraulic radius, and the slope, and, for high stages, a change in the slope affects C to only a small extent. The magnitude of the effect on the slope for any given rate of change in the gage height will depend on the natural slope of the stream, the greater effect resulting on a river with a flat slope, and having medium or low velocities. On streams where the velocities are higher, and the gage is located near rapids, a change in stage does not affect the slope to any appreciable extent.

There are two cases where the effect of the change in slope has to be considered, *viz.*, (*a*) in the correction of discharge measurements made during periods of changing stage, and (*b*) in determining the daily discharge from the rating curve.

194. Two Methods for Correcting Discharges Made under Changing Stages.—Since the relation of the discharge and the gage height will be governed by the slope in each of the above cases, it is necessary that the slope of the stream be known for any stage under constant flow as well as the slope during any period of rapidly increasing or decreasing flow. This determination of the slope under constant flow can be made at various stages and a slope curve drawn to show the relation between slope and gage height. The slope during periods of changing stages may be determined by means of two gages set at a common datum, and a suitable distance apart or the increase in slope over the natural slope may be calculated and applied to the natural slope of the stream. Knowing the natural slope and the varying slope at any stage, the true discharge may be obtained on the assumption that the discharges vary as the square root of the slope.

The first method is that which is given in *U. S. Water Supply Paper* 345. It is assumed that the discharge varies with the square root of the slope and that the hydraulic radii of sections

between the two gages used in determining the slope are approximately equal. Then if H_1 equals the observed difference in elevation of the two gages, Q_1, the measured discharge at that time, H_n the *normal* difference in elevation, determined by averaging the daily record of differences extending over a period of time and Q_n equals the corresponding *normal* discharge, the following relation may be shown to be true:

$$\frac{Q_1}{Q_n} = \sqrt{\frac{H_1}{H_n}}.$$

By first determining the *normal* difference in elevation between the two gages, a *normal* rating curve can be constructed

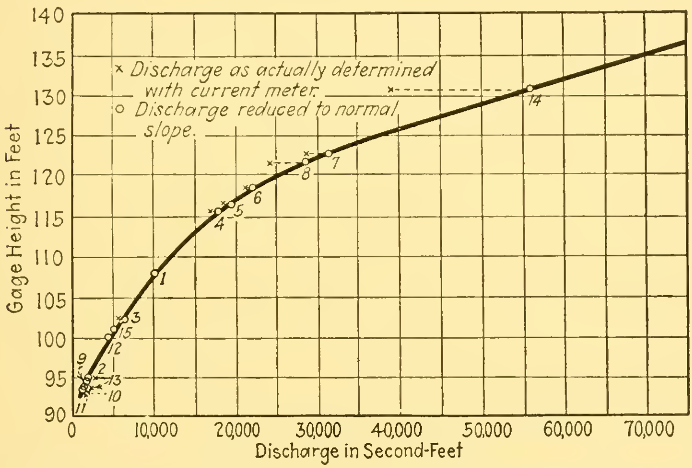


FIG. 125.—Rating curve for Yazoo River at Greenwood, Miss. (*From U. S. Water Supply Paper* 345.)

from a series of discharge measurements, observing the discharge, gage height, and difference in readings of the two gages. Then having H_1 and Q_1 by measurement, and H_n by computation, Q_n is readily obtained for each measurement. The several values of Q_n may then be plotted against their corresponding gage heights and a curve drawn through the plotted points. This curve will constitute the *normal* rating curve. From the rating curve a rating table may be made for greater convenience.

This curve or table is used by first observing the gage height and difference in gage readings. With the observed gage height the *normal* discharge may be read from the rating curve or

taken from the rating table. With the difference in gage heights the factor $\sqrt{\frac{H_1}{H_n}}$ may be determined either by computation or from a table of $\sqrt{\frac{H_1}{H_n}}$. The product of Q_n and $\sqrt{\frac{H_1}{H_n}}$ will give the true discharge.

For most reliable results it is necessary that the two gages be set at exactly the same datum and that the water surface between them be smooth and free from ripples, etc. One of the gages, preferably the lower, should be the gage used for determining the discharge from the rating curve.

Figure 125 is the *normal* rating curve for the Yazoo River at Greenwood, Miss. This curve was constructed from data given in the table in Fig. 126. The section is located at the gage at

Date	Hydrographer	Width, feet	Area of section, sq. ft.	Mean velocity, ft. per sec.	Gage height, feet	Actual discharge, sec.-ft.	Philipp gage height, feet	Fall, feet	$\frac{H_1}{H_n}$ z	$\sqrt{z}$ $\sqrt{\frac{H_1}{H_n}}$	Normal discharge, sec.-ft.	Measurement No.
1908												
June 17	M. R. Hall	307	5,220	1.95	107.84	10,200	125.0	17.2	1.012	1.006	10,100	1
Dec. 5	W. A. Lamb	252	1,340	1.43	94.90	1,920	115.3	20.4	1.200	1.095	1,750	2
9	do	300	3,380	1.85	102.35	6,260	118.5	16.2	0.953	0.976	6,410	3
1909												
Feb. 27	do	411	7,860	2.19	115.60	17,100	131.1	15.5	.912	.955	17,900	4
Mar. 4	do	416	8,270	2.30	116.55	19,000	132.8	16.2	.953	.976	19,500	5
12	Tallahatchie Drainage Commission engineers.	660	9,690	2.26	118.51	21,900	135.0	16.5	.971	.985	22,200	6
27	do	662	13,300	2.15	122.72	28,600	136.8	14.1	.829	.910	31,400	7
Apr. 3	do	659	11,500	2.11	121.65	24,300	133 9	12.3	.724	.851	28,600	8
Aug. 24	E. H. Swett	260	1,540	1.10	94.67	1,690	113.0	18.4	1.082	1.040	1,620	9
26	do	259	1,430	1.01	94.28	1,450	112.7	18.4	1.082	1.040	1,390	10
27	do	258	1,400	1.00	94.16	1,410	112.5	18.3	1.076	1.037	1,360	11
1911												
Aug. 21	R. E. Robertson	305	2,940	1.57	100.12	4,620	119.5	19.4	1.141	1.068	4,330	12
Nov. 18	do	269	1,290	1.19	94.30	1,540	112.9	18.6	1.094	1.046	1,470	13
1912												
Apr. 5	W. E. Hall	663	17,300	2.26	130.64	39,100	138.9	8.3	.488	.699	55,900	14
Sept. 25	do	285	3,030	1.67	101.16	5,070	118.9	17.7	1.041	1.020	4,970	15

FIG. 126.—Discharge measurements of Yazoo River at Greenwood, Miss., in 1908–1912. (*From U. S. Water Supply Paper* 345.)

Greenwood. The second gage was located upstream at Philipp. Data were obtained over a period extending from June, 1908 to September, 1912. The *normal* discharge was obtained as indicated by the relation that $Q_n = Q_1 \div \sqrt{\frac{H_1}{H_n}}$.

As an illustration of its use, take the readings for Apr. 12, 1912 (Fig. 127). The gage at Greenwood read 130 ft.; that at Philipp, 138.6 ft. The normal difference in the two gages was 17.0 ft. as obtained from a four-year record. Computation:

From the curve, gage $= 130.0'$, $Q_n = 54{,}000$ cu. ft. per second.

$$H_1 = 138.6' - 130.0' = 8.6'.$$
$$H_n = 17.0'.$$
$$\frac{H_1}{H_n} = \frac{8.6}{17.0} = 0.505.$$
$$\sqrt{\frac{H_1}{H_n}} = 0.711 \ (= \sqrt{z} \text{ in table in Fig. 127}).$$
$$\begin{aligned} Q_1 &= Q_n \times \sqrt{\frac{H_1}{H_n}} \\ &= 54{,}000 \times .71 \\ &= 38.400 \text{ cu. ft. per second,} \end{aligned}$$

which was the true discharge on that day.

Another method for correcting for changing state is that given in *U. S. Water Supply Paper* 375-*E*. This method is based on the same assumption that the discharge will vary with the square root of the slope. If S_1 and Q_1 represent the slope and discharge at any constant stage, and S_2 and Q_2 the slope and discharge at the same stage but at a time when the stage is rapidly changing so that the slope is different from what it would be under constant flow, the relation

$$\frac{Q_1}{Q_2} = \sqrt{\frac{S_1}{S_2}} \text{ will be true.}$$

The slope S_1 will be obtained from a curve of slopes *vs.* gage heights, constructed from a series of slopes and gage heights obtained during periods of constant flow. S_2 is composed of two parts, (*a*) the normal slope S_1 and (*b*) the change in slope due to the change in velocity. This second part may be shown to be equal to the ratio of the rate of change of stage to the velocity of advance of the new slope. We may, therefore write

$$\frac{Q_1}{Q_2} = \frac{\sqrt{S_1}}{\sqrt{S_1 + \dfrac{\text{rate of change of stage}}{\text{velocity}}}}.$$

The use of this relation requires a determination of the rate of change of stage and the velocity of advance of the new slope. The rate of change can be computed from hourly observations of the change or from the record of an automatic water level

Day	Gage height			$\sqrt{z}$	"Normal" discharge, Qn, second-feet	True discharge $Qn\sqrt{z}$ second-feet
	Philipp, feet	Greenwood, feet	Difference or fall, feet			
1	138.6	129.9	8.7	0.716	53,700	38,400
2	138.7	130.2	8.5	.707	54,700	38,700
3	138.8	130.3	8.5	.707	55,000	38,900
4	138.9	130.5	8.4	.703	55,800	39,200
5	139.0	130.6	8.4	.703	56,100	39,400
6	139.0	130.7	8.3	.699	56,400	39,400
7	139.0	130.7	8.3	.699	56,400	39,400
8	139.0	130.6	8.4	.703	56,100	39,400
9	138.8	130.5	8.3	.699	55,800	39,000
10	138.8	130.3	8.5	.707	55,000	38,900
11	138.7	130.2	8.5	.707	54,700	38,700
12	138.6	130.0	8.6	.711	54,000	38,400
13	138.6	129.8	8.8	.720	53,300	38,400
14	138.4	129.6	8.8	.720	52,600	37,900
15	138.4	129.5	8.9	.724	52,300	37,900
16	138.3	129.7	8.6	.711	53,000	37,700
17	138.4	130.4	8.0	.686	55,400	38,000
18	138.4	130.0	8.4	.703	54,000	38,000
19	138.4	129.9	8.5	.707	53,700	38,000
20	138.3	129.9	8.4	.703	53,700	37,800
21	138.3	129.9	8.4	.703	53,700	37,800
22	138.2	129.8	8.4	.703	53,300	37,500
23	138.0	129.6	8.4	.703	52,600	37,000
24	137.9	129.4	8.5	.707	52,000	36,800
25	137.8	129.1	8.7	.716	50,900	36,500
26	137.8	128.8	9.0	.727	49,900	36,300
27	137.6	128.6	9.0	.727	49,300	35,800
28	137.5	128.4	9.1	.731	48,600	35,500
29	137.5	128.4	9.1	.731	48,600	35,500
30	137.4	128.4	9.0	.727	48,600	35,300
Total.......	....	....	257.4	....	....	1,135,500
Mean.........	....	....	8.6	....	....	37,800
Maximum.....	139.0	130.7	9.1	....	....	39,400
Minimum.....	137.4	128.4	8.0	....	....	35,300

FIG. 127.—Slope method of computing discharge of Yazoo River at Greenwood, Miss., for month of April, 1912. (*From U. S. Water Supply Paper* 345.)

recorder. The velocity of advance of the new slope will be the same as the surface velocity. The surface velocity may be obtained by dividing the mean velocity by the coefficient 0.9 for large streams and 0.85 for small ones.

Finally, the relation may be thus expressed:

$$\frac{Q_1}{Q_2} = \frac{\sqrt{S_1}}{\sqrt{S_1 + \frac{C_s - \Delta H}{V_m}}},$$

where Q_1, Q_2, and S_1 are used as before,
and C_s = reduction coefficient.
ΔH = change in stage per second.
V_m = mean velocity of flow.

The application of this method is illustrated by the following example taken from *U. S. Water Supply Paper* 375-*E*.

Data:

Mean gage height = 28.2 ft.

Corresponding discharge at constant stage (from rating table = 205,000 cu. ft. per second = Q_1.

Area (from area curve) = 38.890 sq. ft.

$C_s = 0.90$.

$S_1 = 0.0001135$.

Observed rate of change in stage = +0.68 ft. per hour.

Computation, first trial:

Mean velocity $= \frac{205,000}{38,890} = 5.27$ ft. per second $= V_m$.

Change in stage per second $= +\frac{0.68}{2600} = +0.000189 = \Delta H$.

By formula

$$\frac{Q_1}{Q_2} = \frac{\sqrt{S_1}}{\sqrt{S_1 + \frac{C_s \Delta H}{V_m}}}.$$

Substituting,

$$\frac{205,000}{Q_2} = \frac{\sqrt{0.0001135}}{\sqrt{0.0001135 + \frac{0.9 \times .000189}{5.27}}}.$$

$$\therefore Q_2 = 232,000 \text{ cu. ft. per second.}$$

Second trial:

$$\frac{232,000}{38,890} = 5.97 = V_m.$$

$$Q_1 = 205,000.$$

$$S_1 = 0.0001135.$$

$$\Delta H = 0.000189.$$

Substituting,

$$\frac{205,000}{Q_2} = \frac{\sqrt{0.0001135}}{\sqrt{0.0001135 + \frac{0.9 \times 0.000189}{5.97}}}.$$

$$Q_2 = 229,000 \text{ cu. ft. per second.}$$

The corresponding current meter measurement was 229,200 cu. ft. per second.

CHAPTER XIV

EFFECTS OF ICE ON STREAM FLOW

195. Occurrence of Ice.—The streams in the northern part of the United States are generally covered with ice during four or five months each year, and even states included in the Central and Atlantic groups will have their streams closed by ice for a few weeks. It is not often, however, that a stream of any size will freeze over throughout its entire length, there being certain sections which will remain open. The principal variables which govern the freezing of the water are the temperature and the velocity of the water. When the temperature reaches 32° and the stream has a low velocity, the water will freeze over. At rapids and stretches where the bed is quite rough and the water is agitated, the ice cover will not be formed, but a form of ice known as needle ice will be formed. Back of dams or in other stretches where there is still water, ice cover will form very rapidly. Where there are springs present, or near outlets from small streams, the temperature of the water may be raised by the inflowing water to such a point that freezing will not occur.

The following table taken from *U. S. Water Supply Paper* 187 gives a summary of the water conditions at 179 current meter stations.

TABLE XIII.—SUMMARY OF WINTER CONDITIONS AT CURRENT METER STATIONS

Class	New England	New York and Lower Michigan	Atlantic states	Central states
1. Smooth permanent ice cover.....	11	11	13	35
2. Tendency for anchor and needle ice to accumulate.............	4	5	2	
3. Unstable ice cover...............	10	3	21	9
4. Rough ice cover and piling up....	7	3	6	
5. Tendency for ice jams...........	1	1	8	
6. Remain open....................	5	10	7	7

Another table, taken from *U. S. Water Supply Paper* 187, shows the duration of ice cover in different areas where streams freeze during the winter.

TABLE XIV.—DURATION OF ICE COVER, BY AREAS

Locality	Date of closing in	Date of breaking up	Time frozen, months
Northern Maine	Nov. 15 to 30	Mar. 15 to Apr. 15	3½ to 5
Northern Michigan	Dec. 1 to 30	Mar. 1 to 15	2 to 3½
Lower New England	Dec. 15 to Jan. 1	Feb. 15 to Mar. 15	1½ to 3
New York	Dec. 1 to 15	Mar. 15 to 31	3 to 4
Pennsylvania	Jan. 1 to 15	Mar. 1 to 15	1½ to 2½
Illinois	Dec. 15 to Jan. 15	Feb. 15 to Mar. 1	1 to 2½
North Dakota	Nov. 15 to 30	Mar. 15 to 31	3½ to 4½

Other tables showing the duration of the ice season may be found in *U. S. Water Supply Papers* 187 and 337.

The foregoing will serve to emphasize the fact that there is a considerable number of streams in the United States where the effect of ice on the flow must be considered.

196. Kinds of Ice Found in Rivers.—Before studying the effect of ice on the flow of a stream, the several forms of ice which are found in a stream should be considered because each has a different effect on the flow conditions of the stream. The three kinds of ice which are formed are (*a*) surface ice, (*b*) frazil ice, and (*c*) anchor ice.

197. Surface Ice.—Surface ice is formed by the cooling of the surface layer of water to the freezing point. The water along the edges begins to freeze first and the freezing extends towards the center, meanwhile becoming deeper. As previously noted, surface ice does not form as readily when the water has velocity as when the water is quiet, due in the first case to the agitation of the water and the movement of the warmer bottom layers of water to the surface.

198. Frazil Ice.—Frazil ice consists of fine elongated needles, cubical crystals, and plates. It is formed when the temperature reaches 32° and the velocity of the water is such as to prevent the formation of surface ice. Frazil ice is never formed under a cover of ice but is transported by the stream in a floating mass

of slush. It is easily visible only when there are large quantities present, otherwise it is hard to detect.

199. Anchor Ice.—Anchor ice resembles frazil ice in many respects but differs completely in its manner of formation. It forms in large quantities on the beds of rivers or on objects under the water surface. It forms most quickly on dark-colored rocks.

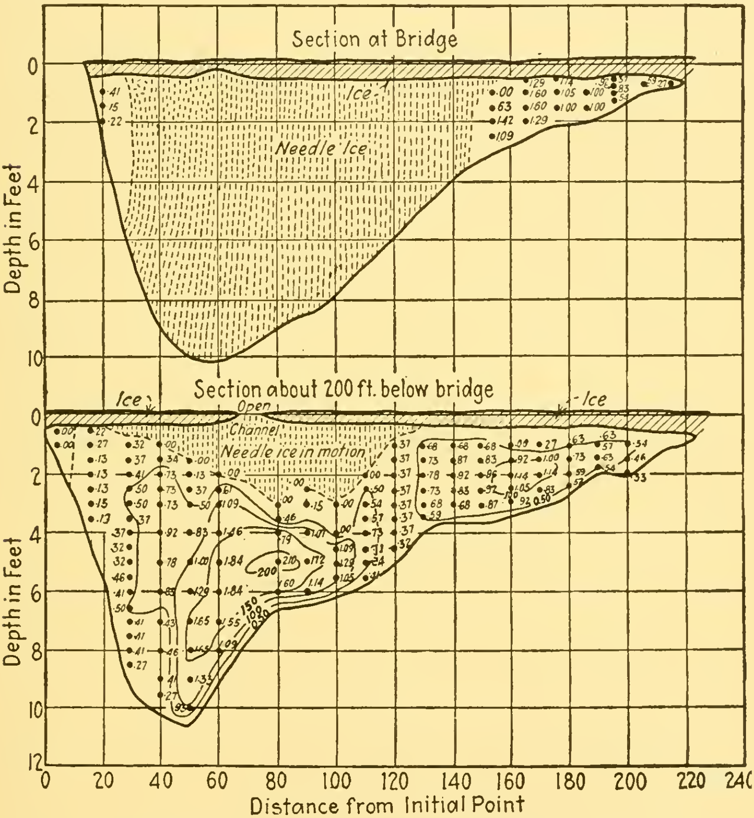


Fig. 128.—Cross-sections of Chemung River at Chemung, N. Y., showing effect of needle ice. (*From U. S. Water Supply Paper* 187.)

The theory of its formation is that it is due to the transmission of heat by radiation. Very often during the day the sun's rays will cause the anchor ice to become detached from the rocks and other obstacles and rise to the surface and combine with any frazil ice that may be present.

200. Effect of Ice Cover on Stage-discharge Relation.—There are two changes in the conditions existing at the control section

which occur when an ice cover is formed. They are (*a*) the replacing of the air on the surface of the water by the ice which increases the frictional resistance to flow, for the friction between the flowing water and the ice is greater than that between flowing water and air; and (*b*) the reduction of the cross-sectional area because of the displacement of a portion of the water by ice. Consequently, if the flow remains unchanged one of two things must result; either the slope of the river must increase or the

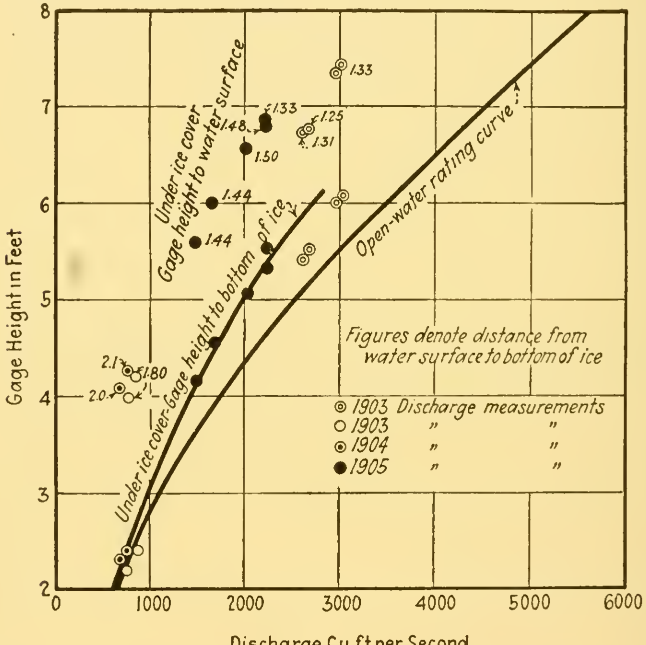


FIG. 129.—Rating curve under ice cover, Connecticut River at Orford, N. H. (*From U. S. Water Supply Paper* 187.)

cross-section must increase. With the presence of the ice cover, it is impossible for any very appreciable change to occur in the slope, and consequently the general effect is to increase the stage.

In general, a given flow will register a higher gage reading under ice-cover conditions than under open-water conditions, the amount of difference depending upon the amount of ice present at the control section. Consequently, all discharge measurements will plot to the left of the open water rating curve. In

some instances the conditions may be such that the measurements will plot on the curve but under no circumstances will they plot to the right of the curve (Fig. 129).

Where the ice is not in flotation at the control, but is bridged across, and is not in contact with the water surface, the open water stage-discharge relation will not be destroyed. In the case of ice jams forming between the gage and the control or below the control and causing the control to be drowned out, a new control will be formed by the jam of ice.

201. Effect of Ice Cover on the Distribution of Velocities in the Vertical.—In Chap. III it was observed that the distribution of velocities in a vertical for open water conditions followed a parabolic law, the actual values of velocities at the different points in the vertical being affected by the resistance to flow met with by the water at the different depths. The maximum velocity occurred generally in the upper third; the surface velocity depended upon the width and depth of the channel and roughness of the bed and sides; and the bed velocity depended on the roughness of the bed. With ice cover formed, the same controlling factors are present except that the air surface is replaced by the ice surface. The ice surface being rougher than air, the surface velocity is reduced. This causes a readjustment of velocities throughout the vertical.

A most comprehensive study of the effect of ice cover on the flow of streams was made by H. K. Barrows and R. E. Horton on rivers in New England and New York and serves as an authority on this work.[1]

TABLE XV.—RELATION OF TOP, MAXIMUM AND BOTTOM VELOCITIES FOR OPEN-WATER CONDITIONS[1]

	Velocity in terms of velocity = 1.00	Difference as regards maximum velocity
Top	1.15	0.03
Maximum	1.18	
Bottom	0.50	0.68

[1] *U. S. Water Supply Paper* 187.

This table was made from the results of seventy-eight curves for various stations in New York. It shows the relation of air or water surface friction to that of bed friction to be 0.044.

[1] *U. S. Water Supply Paper* 187.

The following table for smooth ice cover and rough ice cover show the average ratio of the resistance due to ice cover to that due to roughness of the bed to be 0.58 and 1.28 respectively.

Table XVI.—Comparative Effect of Bed and Ice Friction on the Vertical Velocity Curves under Smooth Ice Cover[1]

River and station	Gage height	Number of curves	Average depth	Average velocity	Difference as regards maximum velocity: Top (1)	Bot. (2)	Ratio 1:2
Kennebec, North Anson, Me.....	3.48	18	2.6	1.17	0.22	0.62	0.35
	4.17	19	2.5	1.30	0.30	0.62	0.48
	4.77	9	2.9	1.33	0.40	0.57	0.70
Connecticut, Orford, N. H.......	4.15	18	4.0	1.04	0.50	0.66	0.76
	5.59	7	4.6	1.08	0.27	0.72	0.38
	6.00	7	4.9	1.11	0.32	0.72	0.44
	6.70	21	5.8	1.23	0.41	0.65	0.63
Fish, Wallagrass, Me............	3.91	8	2.5	0.90	0.23	0.51	0.45
	5.04	8	3.6	1.25	0.64	0.74	0.86
Winooski, Peidmont, Vt.........	....	26	1.6	2.12	0.59	0.77	0.77
Wallkill, Newpaltz, N. Y........	....	5	8.0	2.18	0.37	0.65	0.57
Mean......................	....	....	...	....	0.39	0.66	0.58

[1] *U. S. Water Supply Paper* 187.

Table XVII.—Comparative Effect of Bed and Ice Friction on Vertical Velocity Curves under Very Rough Ice Cover[1]

River and station	Number of curves	Average depth	Average velocity	Difference as regards maximum velocity: Top (1)	Bottom (2)	Ratio 1:2
Roundout Creek, Rosendale, N. Y.	4	5.3	0.74	1.02	0.55	1.86
Wallkill, Newpaltz, N. Y..........	4	14.6	2.98	0.81	0.89	0.91
Winooski, Peidmont, Vt...........	5	6.2	1.99	1.11	1.05	1.06
Mean........................	..	....	....	0.98	0.83	1.28

[1] *U. S. Water Supply Paper* 187.

These tables show that the curve will become flatter as the depth increases, the mean velocity remaining constant, and will become greater for a given depth as the velocity is increased.

From the studies carried on by Barrows and Horton, it was found that the vertical velocity curve for ice cover was drawn back in its upper portion on account of the greater retarding effect of ice over that of air. This resulted in there being two

threads of mean velocity, namely, at 0.13 depth and at 0.71 depth, whereas without ice cover there was but one at 0 61 depth. Also, the thread of maximum velocity was lowered from 0.14 depth to 0.36 depth. The bottoms of the two curves were at substantially the same position, as might be expected.

With a very rough ice cover, the ice resistance may affect the curvature of the lower portion of the curve as well as the upper.

Figure 130 taken from *U. S. Water Supply Paper* 187 shows the effect of ice cover on the shape of the curve.

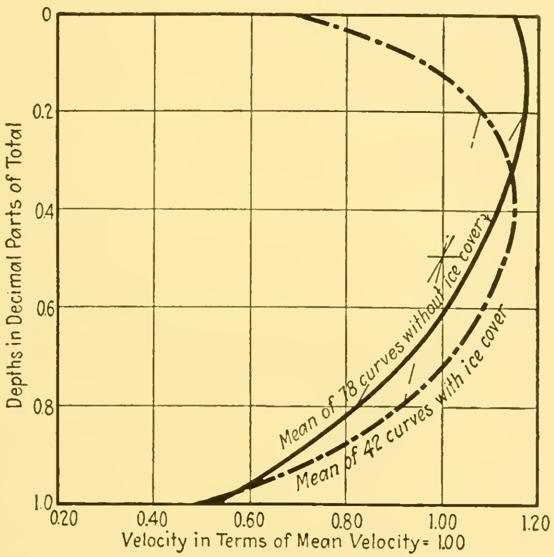


FIG. 130.—Comparison of vertical velocity curves for streams with and without ice cover. (*From U. S. Water Supply Paper* 187.)

202. Position of Threads of Mean Velocity under Ice-cover Conditions.—Barrows and Horton found from a study of 352 vertical velocity curves made under widely different conditions that, in general, under ice cover two threads of mean velocity occurred in the vertical, their average position being at 0.10 and 0.71 below the bottom of the ice. These positions of the thread of mean velocity tend to be lowered as the stage increases. The range in position for both upper and lower threads of mean velocity was 0.18 depth.

203. Position of Maximum Velocity and Relation to Mean Velocity under Ice-cover Conditions.—The average position of maximum velocity was found to be at 0.37 depth below the ice, varying from 0.19 to 0.52 depth. It is lowered as the depth and velocity increase and, hence, as the stage increases. For very rough ice cover, the open water curve may be completely reversed, the slowest velocity occurring at the ice and the maximum velocity below mid-depth.

204. Relation of Average of Velocities at 0.2 and 0.8 Depth to the Mean Velocity.—Studies have shown that the average coefficient for obtaining the mean velocities from the mean of the velocities at 0.2 and 0.8 depth is 1.002, the range being from 0.98 to 1.04. This coefficient seems, in general, to decrease slightly as the gage height increases.

Fig. 131.—Gaging station on Farmington River at New Boston, Mass. during winter months. Showing ice jam at control section. (*Courtesy, U. S. Geological Survey.*)

In the case of the open-water curve, the mean ordinate to the parabola was the mean of the ordinates at 0.21 and 0.79 depth. When the mean of the velocities at 0.2 and 0.8 depth were used there was a slight error introduced which was considered negligible. With the ice-cover curve, the curve is no longer a parabola. However, the studies made of the velocities at the 0.2 and 0.8 depth seemed to indicate that such measurements were reasonably accurate and had the advantage of being convenient.

205. Field Methods for Measuring Flow under Ice-cover Conditions.—Among the more important features for measuring

flow under ice-cover conditions which are described in detail in *U. S. Water Supply Paper* 337, are the following:

The measuring section should have the same general characteristics as good open-water sections; that is, depth and velocity should be

Fig. 132.—Gaging station So. Branch Ashuelot River near Marboro, N. H. during winter months. Complete ice cover has formed at the station. (*Courtesy, U. S. Geological Survey.*)

Fig. 133.—Gaging station on Connecticut River at First Connecticut Lake, near Pittsburg, N. H. during winter months. Open water conditions prevail at control and measuring sections. (*Courtesy, U. S. Geological Survey.*)

fairly uniform, the bottom should be smooth and there should be no cross currents.

If the presence of frazil or of floating anchor ice at a section affects more than about 10 per cent of the total cross-section, the measure-

ments should, if possible, be made at other sections where such conditions do not exist, for example, at rapids where the velocities are high enough to carry away the ice quickly.

FIG. 134.—Gaging station on Moss Brook at Wendell Depot, Mass. during winter months. Ice cover forms at station. (*Courtesy, U. S. Geological Survey.*)

FIG. 135.—Cutting holes in ice at gaging station on Passadumkeag Stream at Lowell, Maine. (*Courtesy, U. S. Geological Survey.*)

The holes in the ice should be spaced from 5 to 10 ft. apart depending on the width of the stream. They should be large enough to permit the meter to be easily raised or lowered. The gage height to the water surface should be read.

At each hole should be recorded (*a*) the thickness of the ice, (*b*) the distance from the under surface of the ice to the water surface, (*c*) the total depth of the water.

From these data, the correct depth at which to hold the meter, in order that it may be at the 0.2, 0.8, or 0.5 position beneath the ice, can be computed.

The computations (Fig. 136) would be:

$$0.2 \text{ depth} = (c - b) \times 0.2 + b.$$
$$0.5 \text{ depth} = (c - b) \times 0.5 + b.$$
$$0.8 \text{ depth} = (c - b) \times 0.8 + b.$$

The mean level of the water surface should be read when the surface is pulsating vertically.

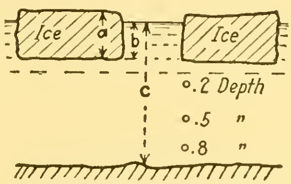


FIG. 136.—Illustrating method for locating measuring points for making velocity observations under ice cover conditions.

For depths under the ice greater than 2.5 ft., the 0.2 and 0.8 method should be used; for depths between 1.5 and 2.5 ft., velocities should be measured at 0.2, 0.5 and 0.8 depth; for depths less than 1.5 ft., the 0.5 method should be used. For this 0.5 depth measurement, the reduction coefficient to obtain the mean velocity should be obtained.

206. Methods for Calculating Stream Flow under Ice-cover Conditions.—Because of the variableness of ice conditions at a station, there is no general method for calculating the stream flow which may be followed in every case. However, there are three methods commonly used which are as follows:

1. Applying the readings of the gage height of the water surface to the open-water rating curve. This method is suited only for the case where there is open water at the control section and there is no backwater at the gage. Any formation of ice between the gage and the control section will not alter the relation between the slope, stage, and discharge, provided the control section remains free from ice.

2. Applying readings of the gage height to a special rating curve based on winter discharge measurements and gage heights to the water surface. This method is suited to cases where the ice conditions remain permanent. The rating curve will not

apply for any transition period from open-water to ice conditions. This method should only be used when many discharge measurements with the ice cover have been found to plot on a smooth curve.

3. Using discharge measurements and data on weather conditions, such as the temperature and precipitation, together with the occurrence of ice in connection with observed gage heights. With this information and the estimated daily discharge for the periods between times of measurement, the probable flow of the stream can be determined. The accuracy of the method will

DEPARTMENT OF THE INTERIOR
UNITED STATES GEOLOGICAL SURVEY
WATER RESOURCES BRANCH

Date.................................. 192 No. of Meas. ....

CURRENT METER NOTES—ICE COVER

________ River, at / Creek, near ________

Dist. from initial point	Thickness of ice / W. S. to bot. ice	Total depth water / Effective depth	Depth of meter below water surface	Rev-olu-tions	Time in seconds	VELOCITY At point	VELOCITY Mean in vertical	VELOCITY Mean in section	Area	Mean depth	Width	Discharge
Totals												

No. ______ of ______ Sheets. Comp. by ____________ Chk. by ____________ Make notes on back.

Fig. 137.—Form used by the U. S. Geological Survey for keeping current meter notes when measuring velocities under ice cover conditions.

depend largely on the uniformity of the stream flow between measurements, the frequency of the measurements, and the experience of the engineer.

This method is known as the "eye method" because of the manner of inspection. Another method involving the several factors considered in the eye method is the "graphic method" which, as the name suggests, requires the construction of curves for study of the winter flow.

Each of these above methods is more thoroughly explained by W. G. Hoyt in *U. S. Water Supply Paper* 337, and reference may be made to it for further details.

APPENDIX

PROBLEMS

1. A river gage is fixed at an inclination of 30 deg. from the horizontal but is graduated to read vertical distances. What should be the actual length of scale per foot of vertical rise?

2. The angle observed with a transit from one end of a 200-ft. base line on a river bank to a sounding taken upon a perpendicular range at the opposite end of the base line was 35 deg. 37 min. What was the distance of sounding from base? What error in computed position of sounding would result from an error of 3 min. in observed angle?

3. Observations were taken at a certain point in a river as follows:

Apr. 1.	Sounding, 80.6 ft.
	Gage height, 35.4 ft.
Apr. 7.	Velocity observations made, but no sounding taken.
	Gage height, 40.4 ft.
Apr. 15.	Sounding, 94.8 ft.
	Gage height, 46.8 ft.

The observations were all made at about the same hour of day. Compute the probable depth for Apr. 7.

4. Soundings were taken in a stream cross-section from one bank to the other at 2-ft. intervals as follows: 0, 0.91, 1.44, 1.92, 2.26, 2.61, 2.87, 3.00, 2.69, 1.78, and 0 ft. Compute area of cross-section to hundredths of a square foot:

a. By method (2) Art. 103.
b. By method (3) Art. 103.
c. By method (4) Art. 103.
d. By method (5) Art. 103.
e. By method (6) Art. 103.
f. By method (6) Art. 103, supposing soundings taken to nearest tenth of a foot only.
g. By method (6) Art. 103, supposing soundings taken to nearest tenth and at alternate stations only.

5. For the purpose of finding the mean cross-section, soundings were taken on five parallel cross ranges as follows, distances being referred to a base line upon the river bank (all figures in feet):

Range 1		Range 2		Range 3		Range 4		Range 5	
Distance out	Sounding	Distance out	Sounding	Distance out	Sounding	Distance out	Sounding	Distance out	Sounding
10.0	0.0	9.8	0.0	8.4	0.0	9.2	0.0	8.6	0.0
20.2	4.9	14.2	1.3	17.6	2.9	14.2	3.9	17.3	4.0
27.0	7.5	21.2	5.0	22.7	5.0	24.7	7.2	23.9	7.9
32.5	9.0	24.8	6.1	29.1	8.3	29.3	9.1	35.5	10.0
etc.		30.6	7.4	34.2	9.5	40.8	11.3	etc.	
		etc.		etc.		etc.			

Compute, for mean cross-section, the distance out of zero point, and the depth at 20 ft. out from base.

6. A float gage registers 5.545 ft. before a rise in stage, and after the rise, it registers 7.750 ft. The float is 6 in. in diameter and floats at a depth of 1 in. before the change in stage occurs. The counterpoise weighs 1½ lb. and has a specific gravity of 11.3. Before the change, it is 3 ft. above the surface of the water. The wire cable connecting the float and the counterpoise weighs 0.0065 lb. per foot. The force necessary to overcome the friction of the mechanism may be taken as 1.5 oz.

Determine the true gage height after the change in stage has occurred.

7. If the equation of the vertical velocity curve be

$$v' = c + p\sqrt{h},$$

in which v' = velocity at any point in the vertical,
h = height of point above bottom,
c and p = constants,

find an expression for the mean velocity in the vertical, and for the depth at which it occurs.

8. An equation given by Hagen for the vertical velocity curve is

$$y^5 = ph,$$

in which y = velocity at height h above bed.

From this equation find an expression for the mean velocity in the vertical in terms of p and the total depth, t, in the vertical; find also at what proportion of the total depth a velocity equal to the mean velocity is reached; and find the relation between mean velocity in the vertical and surface velocity.

9. In a certain vertical 32 ft. deep, Hagen assumed the vertical velocity curve to be expressed by $v = 2\sqrt[5]{h}$, where h is the distance from bottom. Determine the mean velocity in the vertical if the surface velocity is 4 ft. per second.

10. Assuming vertical velocity curve to be an ordinary parabola with horizontal axis and maximum velocity at surface, show that locus of mean velocity is 0.58 depth.

11. Assuming vertical velocity curve to be an ordinary parabola with horizontal axis and maximum velocity at three-tenths depth, show that locus of mean velocity is at 0.65 depth.

12. In a certain vertical 3.0 ft. deep the mean velocity is 3.95 ft. per second. The maximum velocity is 5.00 ft. per second and occurs at a depth

of 0.5 ft. below the surface. Calculate probable position of thread of mean velocity, assuming parabolic variation of velocity in vertical.

13. Assuming a vertical velocity curve to be a parabola with horizontal axis, what will be the depth of the thread of mean velocity if the maximum velocity occurs at $0.231D$, where D is the total depth?

14. If the surface velocity in Problem 13 is 2.00 ft. per second and the maximum is 2.15 ft. per second, what would be the reduction coefficient to apply to the surface velocity in order to obtain the mean velocity?

15. If a vertical has a surface velocity of 4.04 ft. per second and a maximum velocity of 4.26 ft. per second at 0.21 depth and if the vertical velocity curve is assumed to be a parabola with axis horizontal, what is the mean velocity in the vertical and at what depth does it occur?

16. Soundings from bank to bank at 2-ft. intervals in a stream cross-section are 0, 2.0, 4.0, 4.8, 5.4, 6.0, 5.2, 4.0, 2.8, 1.6, and 0. If the flow was found to be 166 sec.-ft., what was the average velocity at the station?

17. Assuming that floats are to be used in a certain gaging; that the maximum velocity in any vertical will evidently not exceed 4 ft. per second; that an error of 0.01 ft. in measuring the length of run is not unlikely, nor an error of $\frac{2}{5}$ sec. in the timing of individual runs; and that it is desired to limit the relative error in determining the velocity for any run to 2 per cent, find the length of run thus called for.

18. What per cent error would be made in a float measurement if the course was 90 ft. long and an error of 0.01 ft. is made in the total length and the maximum error in timing $\frac{2}{5}$ sec. The probable maximum velocity is 5 ft. per second?

19. A tube float of proper length for giving true mean velocity in vertical crossed the upper range 60 ft. out from the base line on shore and crossed the lower range 10.6 sec. later, 70 ft. out from the base, the normal distance between the ranges being 40 ft. Assuming it to have moved in a straight line, what was its actual velocity? What was its velocity normal to the cross ranges? Which of these velocities should be used in computation of discharge, and for what reason?

20. Some loaded tubes are to be made of hollow tin cylinders weighted at the bottom with lead. They are to be 3 in. in diameter and to project 5 in. above the surface of the water. The lead cylinder projects $\frac{3}{4}$ in. up into the tube at its bottom and has the same diameter as the tube. Find formula for length x of lead in terms of l, the total length of tube and lead. Also find value of x for $l = 10$ ft. Make no allowance for joints, and assume weights as follows:

1. cubic inch water = 0.578 oz.
1-in. length of tin tube = 0.78 oz.
1-in. length of lead cylinder = 46.0 oz.

21. In a certain case the mean depth of the stream along the path of a tube was 10.76 ft. The tube was submerged 10.00 ft., and occupied 20.8 sec. in running over an 80-ft. interval. What was the mean velocity of the water in the vertical plane, corrected by Francis' formula?

22. The following data were obtained in a float measurement made in a rectangular canal 48.6 ft. wide. The length of the course was 80 ft.

Length of tube	Distance from left bank		Time in seconds	Water depth
	Upper boom	Lower boom		
10.0	1.0	1.0	26.2	10.9
10.0	1.8	2.2	24.2	10.9
10.0	4.0	4.5	23.8	10.9
10.0	5.0	5.0	20.6	10.9
10.0	5.8	6.2	20.8	10.6
10.0	7.3	7.8	21.0	10.6
10.0	10.5	10.2	22.0	10.6
10.0	11.8	11.0	20.6	10.4
10.0	12.7	13.2	20.4	10.4
10.0	15.0	14.0	21.4	10.4
10.0	17.0	16.5	21.6	10.4
10.0	22.7	23.0	20.0	10.4
10.0	26.5	26.5	20.2	10.4
10.0	30.8	31.2	19.8	10.8
10.0	33.0	32.0	20.2	10.8
10.0	36.0	36.8	20.2	10.8
10.0	38.0	38.0	21.8	10.8
10.0	41.0	40.0	22.0	10.8
10.0	43.2	42.2	23.0	10.8
10.0	46.5	46.4	23.0	10.8

Determine the rate of discharge, making corrections in tube velocities by means of Francis' formula.

23. The Francis tube correction formula is:

$$C = 1 - 0.116\left[\sqrt{\frac{d - d'}{d}} - 0.1\right]$$

in which d = mean water depth along path of tube.
d' = mean depth of immersion of tube.

What relative length would a tube necessarily have in order to require no correction?

24. Soundings from bank to bank at 1-ft. intervals in a stream cross-section are 0, 1.3, 1.8, 2.2, 2.5, 2.8, 3.0, 3.0, 2.9, 2.7, 2.2, 1.3, 0 ft.; and mean velocities past the same verticals are 0, 1.40, 2.08, 2.67, 3.19, 3.70, 3.95, 3.93, 3.63, 3.00, 2.12, 1.14, 0 ft. per second. Compute the total discharge to three significant figures.

25. During certain meter observation, lasting 3 min., the boat and meter move laterally 100 ft. Current velocity indicated by meter is 4.78 ft. per second. Compute probable true velocity.

26. What is the straight-line equation of a meter which makes 36 revolutions per minute in a current of 0.55 ft. per second velocity, and ninety-six revolutions per minute in a current of 1.32 ft. per second velocity? At what velocity does the equation indicate that the wheel will cease to turn?

27. Compute the theoretic number of revolutions to be made per second in a current of 1 ft. per second velocity by a Fteley meter wheel with helicoidal vanes of the following proportions:

Diameter of wheel .... 3.6 in.
Depth of wheel parallel to axis .... 0.9 in.
Length of arc covered by vane, at exterior circumference .... 1.12 in.

28. If, in a current of 3.15 ft. per second velocity, the meter of problem 26 actually makes 403 revolutions in 100 sec., what is the corresponding number of revolutions per foot of advance?

29. Find the theoretical velocity of water (in feet per second) which will turn a frictionless Fteley water wheel 3.5 revolutions per second if the wheel has the following dimensions:

Diameter of wheel .... 3.9 in.
Depth of wheel parallel to axis .... 1.0 in.
Length of arc covered by vane, at exterior circumference .... 1.43 in.

30. For Price meter 1986, the velocity of twenty revolutions in 40 sec. is 1.26 ft. per second and for eighty revolutions in 50 sec. is 3.54 ft. per second. If the equation of the rating curve for the meter is a straight line of the form $v = aN + b$ in which v = velocity of water in feet per second, and N = revolutions per second, determine the equation for v. At what velocity will the meter cease to turn?

31. If the pitch of the helicoidal vanes of a Fteley current meter wheel be 9 in. and the wheel makes 105 revolutions during an observation lasting 70 sec., what is the theoretical velocity of the current to three significant figures, assuming the wheel frictionless?

32. Assuming the meter rating curve to be a straight line, what would be the equation if it made sixty-two revolutions in 26 sec. when moved with a velocity of 4.2 ft. per second through still water, and ten revolutions in 40.0 seconds when moved with a velocity of 0.41 ft. per second?

33. The rating curve for a certain meter shows the following: When the revolutions per second (N) is equal to 4, the velocity (V) is equal to 8.78 ft. per second, and when $N = 1$, $V = 4$. Determine the equation of the line, having it in the form, $V = mN + b$.

34. Assuming the rating curve of a meter to be a straight line, what inconsistency is shown in the statement that a certain meter makes 36 r.p.m. in a current of 0.35 ft. per second velocity, and 96 r.p.m. in a current of 1.32 ft. per second velocity?

35. Compute the theoretic number of revolutions to be made in 1 min. in a current of 5 ft. per second velocity by a Fteley meter wheel with helicoidal vanes of the following proportions:

Diameter of wheel .... 3.5 in.
Depth of wheel, parallel to axis .... 0.8 in.
Length of arc covered by vane at exterior circumference .... 1.2 in.

36. Assuming that the transverse velocity curve for a rectangular-shaped canal is a parabola having its axis vertical and at the center of the section,

calculate, without constructing a diagram, the discharge of the canal on the basis of Harlacher's method, having given the following data:

Scales:	Velocities................	1 in. = 0.5 ft. per second
	Horizontal distances......	1 in. = 2.0 ft.
	Depths..................	1 in. = 1.0 ft.
Maximum velocity, occurring at center . .		3.0 ft. per second
Velocity at ¼ points..................		2.5 ft. per second
Depth		5.0 ft.
Width		20.0 ft.

Choose K consistent with data given.

37. Data were obtained as follows:

Station	Depth	Mean velocity
0	0.29	0
1	1.09	0.23
2	1.48	0.37
4	1.80	0.62
6	1.94	0.83
8	2.12	0.90
10	2.25	0.89
12	2.65	0.88
14	2.76	0.86
16	2.42	0.70
18	2.42	0.73
20	2.32	0.82
22	2.37	0.75
24	2.23	0.77
26	2.26	0.70
28	2.26	0.70
30	2.13	0.60
32	1.66	0.42
34	1.43	0.31
36	1.53	0.33
38	1.40	0.19
40	0.99	0.08
42	0.61	0.01
43	0.34	0

a. Make plots of cross-section and transverse velocity curve choosing convenient scales.

b. Construct Harlacher's diagram.

c. Determine the discharge by means of the diagram.

38. Given a channel with a triangular cross section which has an area of 80 sq. ft. when there are 8 ft. of water in it. If an area curve were drawn for it what would be the slope (as actually measured on the plot) of a tangent drawn to it at a gage height of 8 ft.?

Scale of areas..........................	1″ = 10 sq. ft.
Scale of gage heights....................	1″ = 1 ft.

Answer may be expressed as the tangent of the angle which the tangent line makes with the horizontal.

39. At a certain gaging station the following data was obtained:

Gage	Width	Area	Q
32	320	6,825	32,900
29	305	5,875	26,900
26	288	5,000	21,700
24	277	4,475	18,000
21	259	3,625	14,200
18	240	2,875	10,400
16	226	2,400	8,360
13	204	1,750	5,340
10	179	1,200	3,200
8	160	900	2,000
4	113	300	500
0	0	0	0

(*a*)—Construct an area curve for the station, first drawing tangents through plotted points before drawing in curve.

Use scales, gage: 1 in. = 5 ft. Area: 1 in. = 1000 sq. ft.

(*b*)—Construct a curve showing variation of $A\sqrt{d}$ with gage.

Use scales, $A\sqrt{d}$: 1 in. = 5000 ft. Gage: 1 in. = 5 ft.

(*c*)—Construct a discharge curve with gage heights as ordinates and also with $A\sqrt{d}$ as ordinates.

Use scales, for Q 1 in. = 5000 ft. per second

(*d*)—Compare Q's found for gages 5, 10, 15, 20, 25, 30.

40. Accurate measurements at a certain gaging station furnished the following data:

Gage	Width	Area	Velocity	Discharge
4	327	750	1.84	1,380
8	369	2,040	2.60	5,300
18	533	6,610	3.90	25,800
30	670	13,800		

Estimate by *two reliable methods* the discharge when the stage is 30 ft.

41. At the station referred to in the preceding problem the lower part of the bed is rather shifting, but a long series of measurements has shown an average rating curve to pass through all the above points. The results of three gagings during freshet were as follows:

Date	Gage	Discharge
Apr. 17	6.4	3,000
Apr. 21	10.0	10,000
Apr. 26	7.0	4,000

For several hours on Apr. 24 the gage stood at 8.5 ft. Estimate the rate of discharge at that time.

42. For a certain station the relations between gage height (h), area (A), and mean velocity past the cross-section (v) may be expressed closely as follows:

$$A = 80\,h^{1.44}. \qquad V = 1.2\,h^{0.56}.$$

Determine discharge and width of stream when the gage stands at 20 ft. Determine the intersection with the axis of gage heights of the tangent to the discharge curve at gage height 20 ft.

43. In a certain vertical velocity curve, made under conditions of ice cover, the following observations were taken:

Depth 2 ft.	Velocity 2.0 ft. per second
Depth 5 ft.	Velocity 2.5 ft. per second
Depth 8 ft.	Velocity 2.0 ft. per second
Depth 10 ft.	Bottom

Assuming that the vertical velocity curve is a parabola, with horizontal axis at mid depth, passing through the three points of observation,

(*a*) What is the surface velocity?
(*b*) What is the bottom velocity?
(*c*) What is the mean velocity and at what depth does it occur?

44. A rectangular-shaped canal, having a water section 20 ft. wide and 10 ft. deep is excavated through gravel and its sides are lined with rubble masonry. If the slope of the water surface is $\frac{1}{2500}$, find the rate of flow by means of (*a*) Kutter's formula, (*b*) Manning's formula, and (*c*) Bazin's formula.

45. Find the rate of flow, by means of Manning's formula, of a river with tolerably straight alignment, free from rocks, vegetation, etc. with a surface slope of 0.00015 if the mean area of channel section is 5000 sq. ft. The average width is 500 ft. and the hydraulic radius may be taken as equal to the mean depth.

46. A trapezoidal canal having a base width of 15 ft. with side slopes of 2 horizontal to 1 vertical. The lining of the canal is earth free from any large obstructions. The velocity of flow is 2.5 ft. per second and the measured rate of discharge is 200 cu. ft. per second. What is the value of the slope? Use the Manning formula.

47. A sharp-crested rectangular weir is placed across a rectangular channel 15 ft. wide. The crest height is 5 ft., the crest length is 8.0 ft., and the head on the crest is 1.5 ft. Find the rate of discharge over the weir by means of Francis' formula.

48. How high could the crest of a suppressed weir be placed above the bottom of a rectangular canal and not cause the water level to rise above the edge of the canal if the canal is 7 ft. deep and 9 ft. wide, and is carrying water at a depth of 4.5 ft. with a mean velocity of 2.0 ft. per second?

49. A sharp-crested rectangular weir 5.0 ft. long is set in a rectangular channel 5.0 ft. wide and the crest is 5.0 ft. high. If the head on the weir is 0.85 ft., find the discharge by means of Fteley-Stearns and Bazin's formulas.

50. A suppressed weir, 7.5 ft. long, with the crest 4.0 ft. above the bed of the canal has a head on its crest of 1.4 ft. Determine the discharge by the Francis and King formulas (*a*) not correcting for velocity of approach, (*b*) correcting for velocity of approach.

51. Given a triangular shaped weir with a 90-deg. angle, having a head of 0.5 ft., find the rate of discharge over the weir by means of King's formula.

52. A trapezoidal weir having a base of 10 ft., sides slopes 1:1, and a head of 2.0 ft. would discharge at what rate?

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